# Reflections on Four Decades of CFD – A Personal Perspective

Antony Jameson

Aerospace Computing Laboratory
Department of Aeronautics and Astronautics
Stanford University

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Antony Jameson, Phil Roe and Bram van Leer
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# Outline of the Talk

# CFD Past, Present and Future

# Influential People in My Life and Sponsors

| | | |
|---|---|---|
| Father Jacques | St. Edmund's School | Shillong, India |
| Sandy (Mr. Sanderson) | Mowden School | England |
| Dr. Hutton | Winchester College | England |
| Walter Oakeshott | Winchester College | England |
| Ernest Franke | Trinity Hall, Cambridge | England |
| Arthur Shercliff | Cambridge Engineering | England |
| Sir William Hawthorne | Cambridge Engineering | England |
| Len Murray | Trades Union Congress | England |
| Rudy Meyer | Grumman Aerospace | US |
| Grant Hedrick | Grumman Aerospace | US |
| Paul Garabedian | Courant Institute | US |
| Jerry C South Jr. | NASA | US |
| Morton Cooper | ONR | US |
| Spiro Lekoudis | ONR | US |
| Charles Holland | AFOSR | US |
| Fariba Fahroo | AFOSR | US |
| Leland Jameson | NSF | US |

# Acknowledgments (1)

**I am deeply honored to share this Symposium with Phil Roe and Bram Van Leer. Their work has provided the foundations of modern CFD methods, and profoundly altered the evolution of the subject.**

**And I want to take this opportunity to thank Z. J. Wang for organizing the Symposium and Fariba Fahroo for her support.**

# Acknowledgments (2)

**Aside from the contributions of my students to the research of our group, I am specially indebted to them for their assistance in creating papers and presentations in electronic form. In recent years I have been particularly helped in this regard by**

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- **Nawee Butsuntorn**
- **Kui Ou**
- **Andre Chan**
- **Manuel Lopez**
- **David Williams**

# History of CFD in Van Leer's view

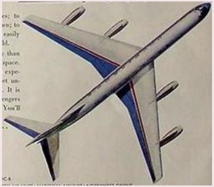
Top level: Jay Boris, Vladimir Kolgan, Bram van Leer, Antony Jameson
Ground level: Richard Courant, Kurt Friedrichs, Hans Lewy, Robert MacCormack, Philip Roe, John von Neumann, Stanley Osher, Amiram Harten, Peter Lax, Sergei Godunov

# Emergence of CFD

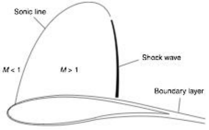
- In 1960 the underlying principles of fluid dynamics and the formulation of the governing equations (potential flow, Euler, RANS) were well established.
- The new element was the emergence of powerful enough computers to make numerical solution possible – to carry this out required new algorithms.
- The emergence of CFD in the 1965 – 2005 period depended on a combination of advances in computer power and algorithms.

**Some significant developments in the 60s:**

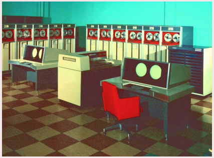
- Birth of commercial jet transport – B707 & DC-8
- Intense interest in transonic drag rise phenomena
- Lack of analytical treatment of transonic aerodynamics
- Birth of supercomputers – CDC6600



DC 8

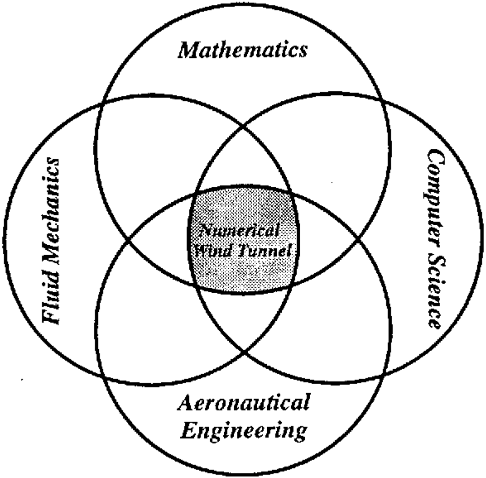



Transonic Flow

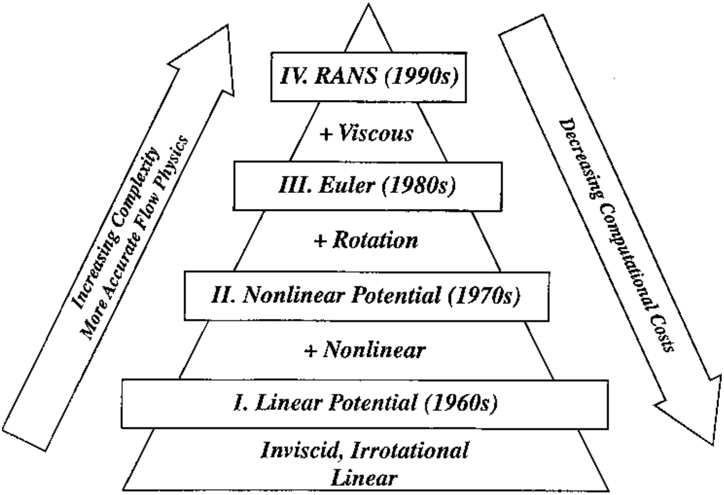


CDC 6600

# Multi-Disciplinary Nature of CFD



## Hierarchy of Governing Equations



# Reflections on the JST Scheme

# Origins of the JST Scheme

The original JST scheme was developed in 1980-81 starting from a code that had been developed at Dornier by Rizzi and Schmidt to solve the Euler equations

This code implemented the MacCormack scheme in finite volume form with additional artificial dissipation to limit oscillations near shocks. It could not converge to a steady state and it appeared from the Stockholm Workshop in 1979 that none of the existing Euler solvers could reach a steady state.

The primary objective of the JST scheme was to solve the steady state problem. This objective was achieved through the use of blended low and high order artificial dissipation and modified Runge-Kutta time stepping with variable local time steps at a fixed CFL number.

Note: The author had been experimenting with Euler solvers since 1976 and had achieved steady state solutions for some simple geometries with the Z scheme. The code EUL1 still exists.

# Original JST Scheme (1980)

The Dornier code (Rizzi-Schmidt) solved for $w$ vol with MacCormack scheme + added diffusion

$$\sim \delta_x \epsilon \delta_x w \text{ vol}, \quad \epsilon \sim \left| \frac{p_{\jmath+1} - 2p_\jmath + p_{\jmath-1}}{p_{\jmath+1} + 2p_\jmath + p_{\jmath-1}} \right|$$

It did not preserve uniform flow on a curvilinear grid.

In order to fix this, move vol outside $\delta_x$.

Then

$$w^{n+1} = w^n - \frac{\Delta t}{\text{vol}}(Q - D), \quad Q = \text{convective terms}$$

For dimensional consistency,

$$D \sim \delta_x \frac{\text{vol}}{\Delta t^*} \delta_x w$$

where $\Delta t^*$ is nominal time step

$$\Delta t^* = \frac{\text{vol}}{(Q + cS)_\imath + (Q + cS)_\jmath}, \quad Q = \vec{q} \cdot \vec{S}$$

Higher order background diffusion was needed for convergence to a steady state. This had to be switched off in the vicinity of a shock to prevent oscillations.

# Design Principles for the JST Scheme

Conservation: integral form

$\Longrightarrow$finite volume scheme

Exact for uniform flow on a curvilinear grid

$\Longrightarrow$constrains discretization, form of diffusion

Steady state independent of $\Delta t$

Eliminates Lax-Wendroff, MacCormack schemes

Concurrent computation

Eliminates LU-SGS schemes $\Longrightarrow$ RK schemes

Non-oscillatory shock capturing

$\Longrightarrow$switched artificial diffusion: upwind biasing

At least second order accurate

$\Longrightarrow$first order diffusion coefficient $\sim \Delta_x p$

Constant total enthalpy in steady flow

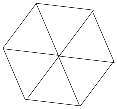
Eliminates Steger-Warming and other splittings

$\Longrightarrow$diffusion for energy equation $\sim \dfrac{\partial}{\partial x}\epsilon\dfrac{\partial}{\partial x}\rho H$

Simplicity

# Mathematical Foundations for the JST Scheme: LED Scheme

A semi-discrete scheme is LOCAL EXTREMUM DIMINISHING (LED) if local maxima cannot increase and local minima cannot decrease.
A scheme in the form



$$\frac{dv_i}{dt} = \sum_{j \neq i} a_{ij}(v_j - v_i)$$

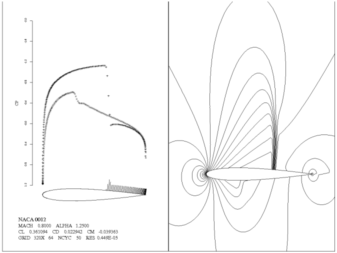
is LED if

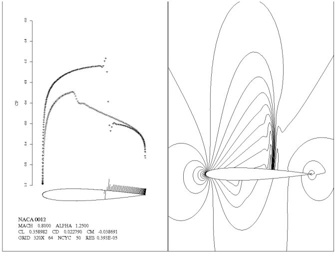
$a_{ij} \geq 0, a_{ij} = 0$ if $i$ and $j$ are not neighbors.
(compact stencil)

In one dimension an LED scheme is total variation diminishing (TVD). With the right switching strategy the JST scheme is LED for scalar conservation laws.

# JST Results

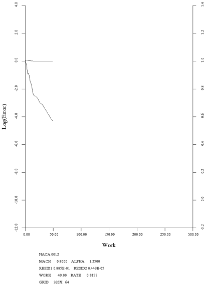
# JST Results for NACA 0012

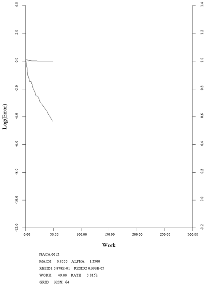



VIS2=1

VIS2=0



# The Quest for a Fast Solver

# The Quest for a Fast Solver

Major aspects of aircraft design such as wing design require solutions of steady state problems.

A fast steady state solver may also be an important ingredient of an implicit scheme for unsteady flow.

# Steady state Solutions and Implicit Schemes

Consider the semi-discrete system

$$\frac{dw}{dt} + R(w) = 0$$

where $R(w)$ is the space residual which results from spatial discretization of the flow equations.

Any implicit scheme, for example the backward Euler scheme

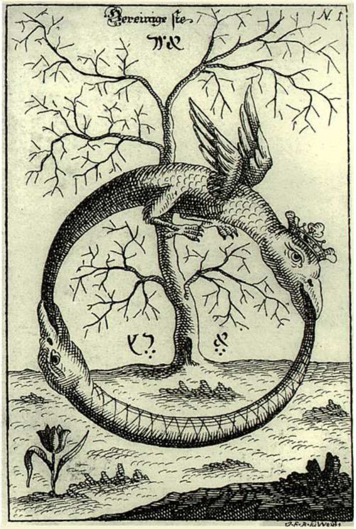
$$w^{n+1} = w^n - \Delta t R\left(w^{n+1}\right)$$

requires the solution of a very large number of coupled nonlinear equations which have the same complexity as the steady state problem

$$R(w) = 0.$$

Accordingly a fast steady state solver is an essential building block for an implicit scheme.

# Paradox



# RANS Example

- In current practice a steady flow over a wing is typically simulated with the Reynolds Averaged Navier-Stokes (RANS) equations on a grid with 10 million cells.
- Using a two-equation turbulence model, this requires the solution of a system of nonlinear equations with $N = 70$ million unknowns.
- Even a linear problem of this size would require iterative solution, considering that direct inversion by Gaussian elimination would require order $(N^3)$ operations.
- By taking advantage of sparsity this might be reduced to order $(N^2)$ with a sophisticated direct solver, but a Newton iteration requiring the solution of a sequence of linear problems of this size would still be very expensive.
- No lower bound for the cost solving steady state problems has been established, but the author believes we should not be satisfied until they can be solved with no more than 100 iterations, each with a cost of order $N$ operations.

# Multigrid Time Stepping Scheme

Towards this goal the author has focused on multigrid time stepping in a full approximation scheme (Jameson 1983)

For the Euler equations this approach has proved successful using

1. Additive Runge-Kutta schemes designed to act as low pass filters (Jameson 1983, 1985)
2. Variations of LU-SGS schemes (Yoon and Jameson 1988, Rieger and Jameson 1988, Jameson and Caughey 2001)

Euler solutions with engineering accuracy can be obtained in about 25 steps with RK schemes, and as few as 5 steps with SGS schemes.

# Additive Runge Kutta schemes with enhanced stability region

To achieve large stability intervals along both axes it pays to treat the convective and dissipative terms in a distinct fashion (Jameson 1985, 1986, Martinelli 1987).

Accordingly the residual is split as

$$\mathbf{R}(\mathbf{w}) = \mathbf{Q}(\mathbf{w}) + \mathbf{D}(\mathbf{w}),$$

where $Q(w)$ is the convective part and $D(w)$ the dissipative part. Denote the time level $n\Delta t$ by a superscript $n$. Then the multistage time stepping scheme is formulated as

$$\begin{aligned}
\mathbf{w}^{(0)} &= \mathbf{w}^{\mathbf{n}} \\
\mathbf{w}^{(1)} &= \mathbf{w}^{\mathbf{0}} - \alpha_1 \Delta t \left( \mathbf{Q}^{(0)} + \mathbf{D}^{(0)} \right) \\
\mathbf{w}^{(2)} &= \mathbf{w}^{\mathbf{0}} - \alpha_2 \Delta t \left( \mathbf{Q}^{(1)} + \mathbf{D}^{(1)} \right) \\
&\dots \\
\mathbf{w}^{(\mathbf{k})} &= \mathbf{w}^{\mathbf{0}} - \alpha_k \Delta t \left( \mathbf{Q}^{(\mathbf{k-1})} + \mathbf{D}^{(\mathbf{k-1})} \right) \\
&\dots \\
\mathbf{w}^{\mathbf{n+1}} &= \mathbf{w}^{(\mathbf{m})},
\end{aligned}$$

where the superscript $k$ denotes the $k$-th stage, $\alpha_m = 1$, and

$$\begin{aligned}
\mathbf{Q}^{(0)} &= \mathbf{Q}\left(\mathbf{w}^{\mathbf{0}}\right), \quad \mathbf{D}^{(0)} = \beta_1 \mathbf{D}\left(\mathbf{w}^{\mathbf{0}}\right) \\
&\dots \\
\mathbf{Q}^{(\mathbf{k})} &= \mathbf{Q}\left(\mathbf{w}^{(\mathbf{k})}\right) \\
\mathbf{D}^{(\mathbf{k})} &= \beta_{k+1} \mathbf{D}\left(\mathbf{w}^{(\mathbf{k})}\right) + (1 - \beta_{k+1}) \mathbf{D}^{(k-1)}.
\end{aligned}$$

# Additive Runge Kutta schemes with enhanced stability region

- The coefficients $\alpha_k$ are chosen to maximize the stability interval along the imaginary axis, and the coefficients $\beta_k$ are chosen to increase the stability interval along the negative real axis.
- These schemes do not fall within the standard framework of Runge-Kutta schemes, and they have much larger stability regions.
- Two particularly effective schemes are:

**4-2 scheme**

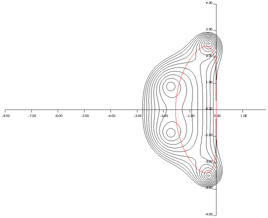
$$\begin{aligned} \alpha_1 &= \tfrac{1}{3} & \beta_1 &= 1.00 \\ \alpha_2 &= \tfrac{4}{15} & \beta_2 &= 0.50 \\ \alpha_3 &= \tfrac{5}{9} & \beta_3 &= 0.00 \\ \alpha_4 &= 1 & \beta_4 &= 0.00 \end{aligned} \tag{1}$$

**5-3 scheme**

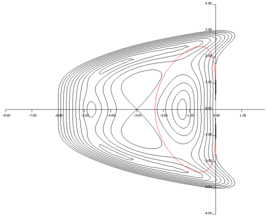
$$\begin{aligned} \alpha_1 &= \tfrac{1}{4} & \beta_1 &= 1.00 \\ \alpha_2 &= \tfrac{1}{6} & \beta_2 &= 0.00 \\ \alpha_3 &= \tfrac{3}{8} & \beta_3 &= 0.56 \\ \alpha_4 &= \tfrac{1}{2} & \beta_4 &= 0.00 \\ \alpha_5 &= 1 & \beta_5 &= 0.44 \end{aligned} \tag{2}$$

- The figures on the next slide display the stability regions for the standard fourth order RK4 scheme and the 4-2 and 5-3 schemes. The expansion of the stability region is apparent.
- The modified schemes have proved to be particularly effective in conjunction with multigrid.

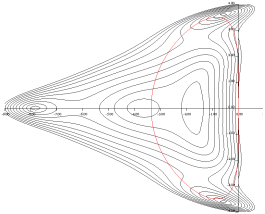
# Additive Runge Kutta schemes with enhanced stability region



CFL 2.4 DIS .04
C1 .25 C2 .3333 C3 .5 C4 1.
NSTAGE 4 NDISSP 4



CFL 2.4 DIS .06
C1 1./3. C2 4./15. C3 5./9. C4 1.
NSTAGE 4 NDISSP 2



CFL 3.5 DIS .06
C1 .25 C2 1/6 C3 .375 C4 0.5 C5 1.
NSTAGE 5 NDISSP 3

# RK-SGS Scheme

RK multigrid schemes are typically augmented by residual averaging (Jameson and Baker 1983) where at each stage the correction $\Delta w$ is smoothed implicitly.

In 1-D

$$-\epsilon\Delta\overline{w}_{i+1}+(1+2\epsilon)\Delta\overline{w}_i-\epsilon\Delta\overline{w}_{i-1}=\Delta w_i$$

and $\Delta\overline{w}$ is used for the stage update.

Rossow (2006) proposed substituting LU-SGS preconditioning sweeps to modify the correction. This concept was further developed by Rossow, Swanson and Turkel (2007). They presented results obtained with 3 and 5 stage RK schemes using 3 LU-SGS sweeps at each stage.

Accordingly the cost of each time step is much greater than that of a standard RK scheme.

# RK-SGS Scheme

During the last year the present author has systematically investigated RK-SGS schemes using an alternate formulation of the LU-SGS preconditioner while exchanging results with Swanson. Two schemes have emerged as best.

1. 2-Stage Additive RK-SGS Scheme

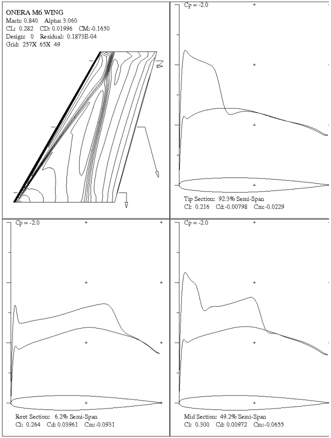
$$\begin{aligned} \alpha_1 &= 0.24 & \beta_1 &= 1.00 \\ \alpha_2 &= 1.0 & \beta_2 &= \tfrac{2}{3} \end{aligned} \tag{3}$$

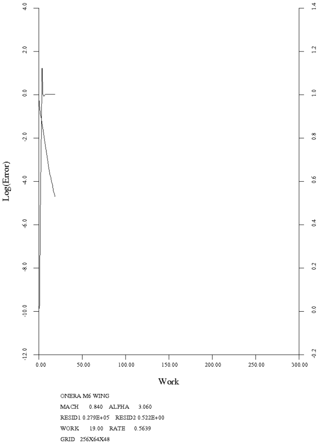
2. 3-Stage Additive RK-SGS Scheme

$$\begin{aligned} \alpha_1 &= 0.15 & \beta_1 &= 1.00 \\ \alpha_2 &= 0.4 & \beta_2 &= 0.5 \\ \alpha_3 &= 1.0 & \beta_3 &= 0.5 \end{aligned} \tag{4}$$

Both schemes have proved robust with a single LU-SGS sweep at each stage, provided that the absolute eigenvalues used in the preconditioner are appropriately bounded away from zero. Hence the computational cost of each time step is quite low.

# Results of RK-SGS Scheme Combined with JST Scheme



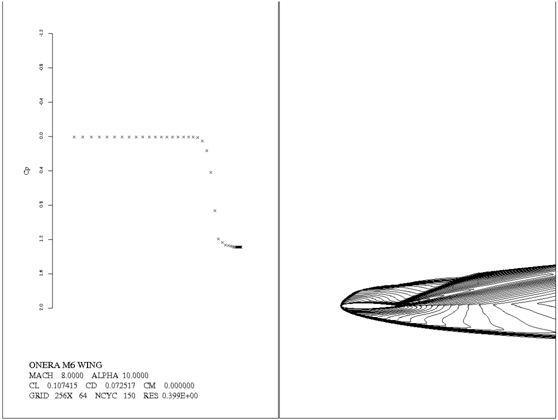

ONERA M6 Wing

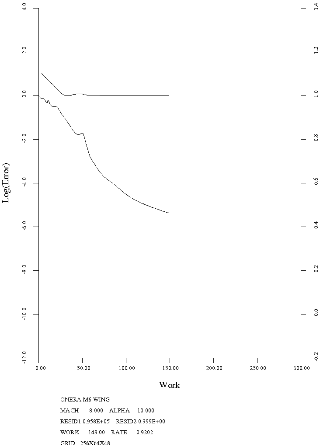
$M = 0.84, \alpha = 3.06, Re = 6 \times 10^6$

5 Digit accuracy of $C_L$ and $C_D$ in 20 steps (Convergence Rate = 0.56)

15 Orders of magnitude reduction of residuals to machine zero in 130 steps (Convergence Rate = 0.77)

# Results of RK-SGS Scheme Combined with JST Scheme




ONERA M6 Wing

$M = 8.0, \alpha = 10.0, Re = 6 \times 10^6$

Solution in 150 steps (Convergence Rate = 0.92)

Needs extra dissipation during the first 80 steps to avoid negative pressure near the wing tip.

# Upwinding with Moving Meshes

# Upwinding with Moving Meshes

With mesh velocity $s$

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial x}(f(w) - sw) = 0$$

- Scheme (1)
  Upwind based on sign of eigenvalues based on relative velocity

$$u - s$$

$$u - s + c$$

$$u - s - c$$

- Scheme (2)
  Upwinding of flux $f(w)$ with absolute eigenvalues

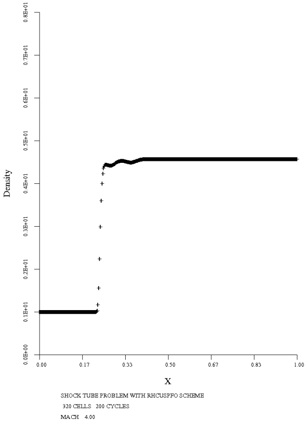
$$u$$

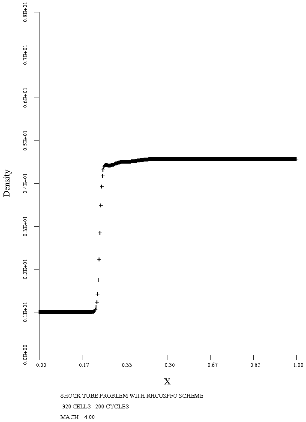
$$u + c$$

$$u - c$$

  separate upwinding of mesh term $sw$ based on sign of $s$

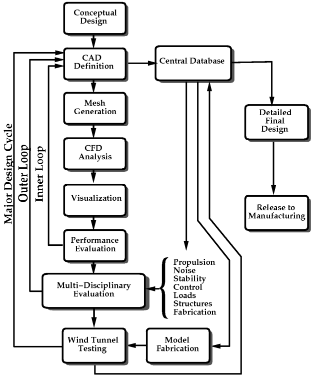
# Shock Tube Problem



# Aerodynamic Design
# &
# Shape Optimization
# via
# Control Theory

# Aerodynamic Design Process




The Aerodynamic Design Process

# Aerodynamic Design Based on Control Theory

- Regard the wing as a device to generate lift (with minimum drag) by controlling the flow
- Apply theory of optimal control of systems governed by PDEs (Lions) with boundary control (the wing shape)
- Merge control theory and CFD
- Find the Frechet derivative (infinite dimensional gradient) of a cost function (performance measure) with respect to the shape by solving the adjoint equation in addition to the flow equation
- Modify the shape in the sense defined by the smoothed gradient
- Repeat until the performance value approaches an optimum

# Aerodynamic Shape Optimization: Gradient Calculation

For the class of aerodynamic optimization problems under consideration, the design space is essentially infinitely dimensional. Suppose that the performance of a system design can be measured by a cost function $I$ which depends on a function $\mathcal{F}(x)$ that describes the shape,where under a variation of the design $\delta\mathcal{F}(x)$, the variation of the cost is $\delta I$. Now suppose that $\delta I$ can be expressed to first order as

$$\delta I = \int \mathcal{G}(x)\delta\mathcal{F}(x)dx$$

where $\mathcal{G}(x)$ is the gradient. Then by setting

$$\delta\mathcal{F}(x) = -\lambda\mathcal{G}(x)$$

one obtains an improvement

$$\delta I = -\lambda \int \mathcal{G}^2(x)dx$$

unless $\mathcal{G}(x) = 0$. Thus the vanishing of the gradient is a necessary condition for a local minimum.

# Aerodynamic Shape Optimization: Gradient Calculation

Computing the gradient of a cost function for a complex system can be a numerically intensive task, especially if the number of design parameters is large and the cost function is an expensive evaluation. The simplest approach to optimization is to define the geometry through a set of design parameters, which may, for example, be the weights $\alpha_i$ applied to a set of shape functions $\mathcal{B}_i(x)$ so that the shape is represented as

$$\mathcal{F}(x) = \sum \alpha_i \mathcal{B}_i(x).$$

Then a cost function $I$ is selected which might be the drag coefficient or the lift to drag ratio; $I$ is regarded as a function of the parameters $\alpha_i$. The sensitivities $\frac{\partial I}{\partial \alpha_i}$ may now be estimated by making a small variation $\delta\alpha_i$ in each design parameter in turn and recalculating the flow to obtain the change in $I$. Then

$$\frac{\partial I}{\partial \alpha_i} \approx \frac{I(\alpha_i + \delta\alpha_i) - I(\alpha_i)}{\delta\alpha_i}.$$

# Symbolic Development of the Adjoint Method

Let $I$ be the **cost** (or **objective**) function

$$I = I(w, \mathcal{F})$$

where

$$\begin{aligned} w &= \text{flow field variables} \\ \mathcal{F} &= \text{grid variables} \end{aligned}$$

The **first variation** of the cost function is

$$\delta I = \frac{\partial I}{\partial w}^T \delta w + \frac{\partial I}{\partial \mathcal{F}}^T \delta \mathcal{F}$$

The **flow field equation** and its **first variation** are

$$R(w, \mathcal{F}) = 0$$

$$\delta R = 0 = \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F}$$

# Symbolic Development of the Adjoint Method (cont.)

Introducing a **Lagrange Multiplier**, $\psi$, and using the **flow field equation** as a **constraint**

$$
\begin{aligned}
\delta I &= \frac{\partial I}{\partial w}^T \delta w + \frac{\partial I}{\partial \mathcal{F}}^T \delta \mathcal{F} - \psi^T \left\{ \left[ \frac{\partial R}{\partial w} \right] \delta w + \left[ \frac{\partial R}{\partial \mathcal{F}} \right] \delta \mathcal{F} \right\} \\
&= \left\{ \frac{\partial I}{\partial w}^T - \psi^T \left[ \frac{\partial R}{\partial w} \right] \right\} \delta w + \left\{ \frac{\partial I}{\partial \mathcal{F}}^T - \psi^T \left[ \frac{\partial R}{\partial \mathcal{F}} \right] \right\} \delta \mathcal{F}
\end{aligned}
$$

By choosing $\psi$ such that it satisfies the **adjoint equation**

$$
\left[ \frac{\partial R}{\partial w} \right]^T \psi = \frac{\partial I}{\partial w},
$$

we have

$$
\delta I = \left\{ \frac{\partial I}{\partial \mathcal{F}}^T - \psi^T \left[ \frac{\partial R}{\partial \mathcal{F}} \right] \right\} \delta \mathcal{F}
$$

This reduces the **gradient** calculation for an arbitrarily large number of design variables at a **single design point** to

$\Longrightarrow$ One Flow Solution + One Adjoint Solution

## Gradient Smoothing

Consider a shape change $f(x) \quad \Longrightarrow \quad f + \delta f$

Set $\delta f = -\lambda g$ to obtain

$$\delta I = \int g \, \delta f \, dx = -\lambda \int g^2 dx$$

A smoothed gradient $\overline{g}$ is defined by

$$\overline{g} - \frac{\partial}{\partial x} \epsilon \frac{\partial \overline{g}}{\partial x} = g$$

and $\overline{g} = 0$ at the end points.

Now set

$$\delta f = -\lambda \overline{g}$$

Then

$$\begin{aligned} \delta I &= -\lambda \int \left( \overline{g} - \frac{\partial}{\partial x} \epsilon \frac{\partial \overline{g}}{\partial x} \right) \overline{g} dx \\ &= -\lambda \int \left( \overline{g}^2 + \epsilon \left( \frac{\partial \overline{g}}{\partial x} \right)^2 \right) \overline{g} dx \end{aligned}$$

Note:

1. If $\overline{g} = 0$ then $g = 0$.
2. The smoothed gradient is the gradient with respect to a Sovolev inner product.

$$< u, v > = \int \left( uv + \epsilon u' v' \right) dx$$

# Constraints

- Fixed $C_L$.
- Fixed span load distribution to present too large $C_L$ on the outboard wing which can lower the buffet margin.
- Fixed wing thickness to prevent an increase in structure weight.

  - - Design changes can be limited to a specific spanwise range of the wing.
  - - Section changes can be limited to a specific chordwise range.

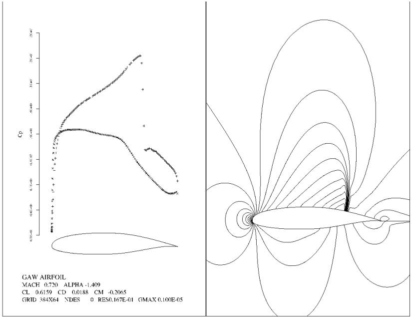
- Smooth curvature variations via the use of Sobolev gradient.

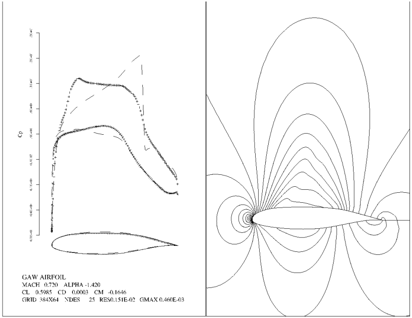
# Shock free airfoil design

- The search for profiles which give shock free transonic flows was the subject of intensive study in the 1965-70 period.
- Morawetz' theorem (1954) states that a shock free transonic flow is an isolated point. Any small perturbation in Mach number, angle of attack, or shape causes a shock to appear in the flow.
- Nieuwland generated shock free profiles by developing solutions in the hodograph plane. The most successful method was that developed by Garabedian and his co-workers. This used complex characteristics to develop solution in the the hodograph plane, which was then mapped to the physical plane. It was hard to find hodograph solutions which mapped to physical realizable closed profiles. It generally took one or two months to produce an acceptable solution.
- By using shape optimization to minimize the drag coefficient at a fixed lift, shock free solutions can be found in less than one minute.

# Two dimensional studies of transonic airfoil design(cont'd)

Pressure distribution and Mach contours for the GAW airfoil



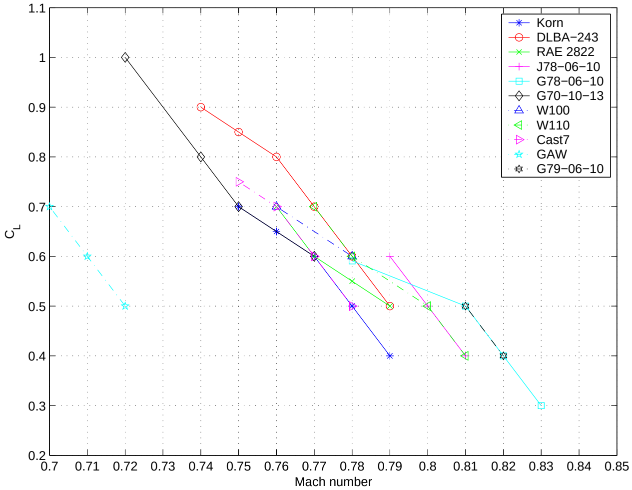




Before the redesign

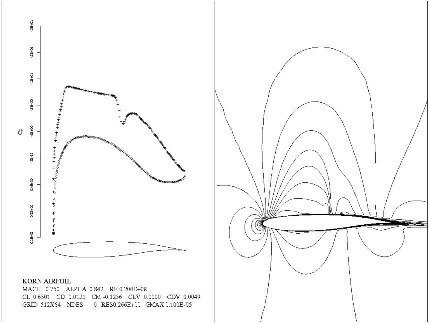
After the redesign

# Two dimensional studies of transonic airfoil design

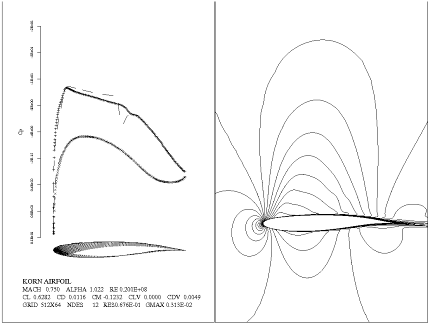
Attainable shock-free solutions for various shape optimized airfoils



# Viscous Korn Airfoil Design

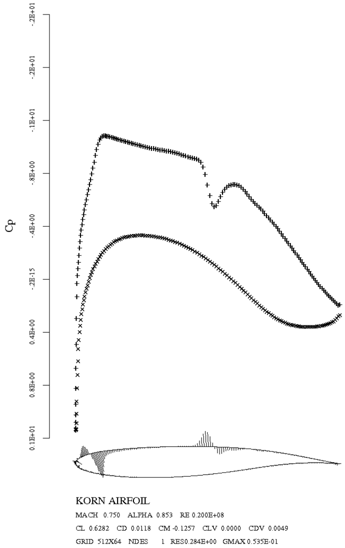



Initial

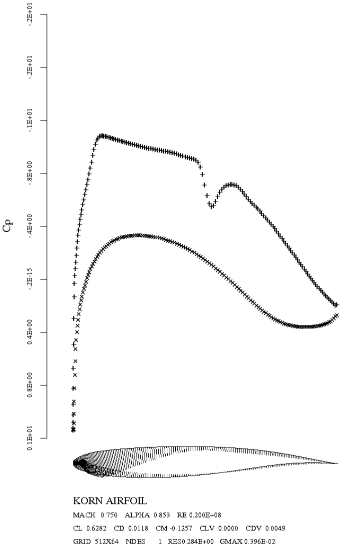



Final

# Viscous Korn Airfoil Design

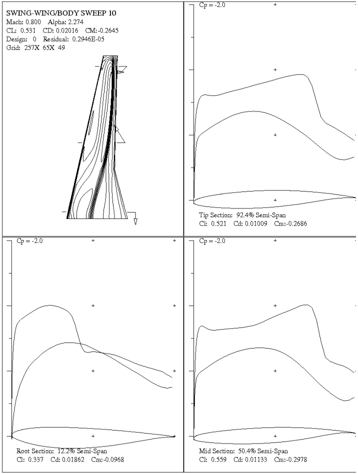



Unsmoothed

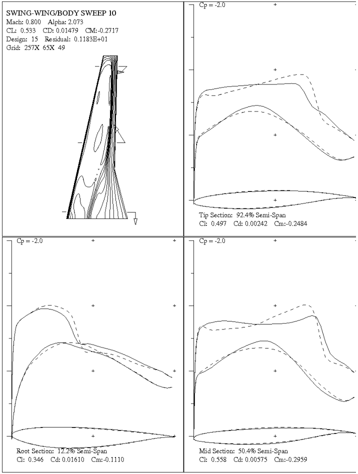



Smoothed

# 3D Redesign of a Deswept Wing Using the New Fast Solver




Initial




Final

# Future Directions

# The Current Status of CFD

- Worldwide commercial and government codes are based on algorithms developed in the 80s and 90s.
- These codes can handle complex geometry but are generally limited to 2nd order accuracy.
- They cannot handle turbulence without modeling.
- Unsteady simulations are very expensive, and questions over accuracy remain.

# CFD Contributions to B787

# CFD Contributions to A380

# The Future of CFD

**CFD has been on a plateau for the past 15 years.**

- Representations of current state of the art:
  - Formula 1 cars
  - Complete aircrafts
- The majority of current CFD methods are not adequate for vortex dominated and transitional flows:
  - Rotorcraft
  - High-lift systems
  - Formation flying
- In order to address these currently intractable problems we need to move towards higher fidelity simulations with large eddy simulation (LES), or ultimately direct numerical simulation (DNS).

# Large Eddy Simulation

The number of DoF for an LES of turbulent flow over an airfoil scales as $Re_c^{1.8}$ **(resp. $Re_c^{0.4}$) if the inner layer is resolved (resp. modeled)**

**Rapid advances in computer hardware should make LES feasible within the foreseeable future for industrial problems at high Reynolds numbers. To realize this goal requires**

- High-order algorithms for unstructured meshes (complex geometries)
- Sub-Grid Scale models applicable to wall bounded flows
- Massively parallel implementation

# High Order Methods

At the Stanford Aerospace Computing Laboratory we have been focusing on the flux reconstruction mehtod first proposed by H. T. Huynh (2007), which provides a unifying framework for a variety of methods.

# Recent Publications from the Stanford Aerospace Computing Laboratory on High Order Methods


1. Castonguay, P., D. Williams, P. Vincent, M. Lopez, and A. Jameson (2011). On the development of a high-order, multi-GPU enabled, compressible viscous flow solver for mixed grids. AIAA P., vol. 2011-3229
2. Jameson, A. (2010). A proof of the stability of the spectral difference method for all orders of accuracy. J. Sci. Comput., vol. 45(1)
3. Jameson, A. (2011). Advances in bringing high-order methods to practical applications in computational fluid dynamics. AIAA P., vol. 2011-3226
4. Ou, K. and A. Jameson (2011). Unsteady adjoint method for the optimal control of advection and Burgers equations using high-order spectral difference method. AIAA P., vol. 2011- 24
5. Vincent, P. and A. Jameson (2011). Facilitating the adoption of unstructured high-order methods amongst a wider community of fluid dynamicists. Math. Model. Nat. Phenom., vol. 6(3)
6. Vincent, P., P. Castonguay, and A. Jameson (2010). A new class of high-order energy stable flux reconstruction schemes. J. Sci. Comput., vol. 47(1)
7. Vincent, P., P. Castonguay, and A. Jameson (2011). Insights from von Neumann analysis of high-order flux reconstruction schemes. J. Comput. Phys., vol. 230(22)
8. Williams, D., P. Castonguay, P. Vincent, and A. Jameson (2011). An extension of energy stable flux reconstruction to unsteady, non-linear, viscous problems on mixed grids. AIAA P., vol. 2011-3405
9. Lodato, G., P. Castonguay, and A. Jameson, Structural LES modeling with high-order spectral difference schemes. In Annual Research Briefs (Center for Turbulence Research, Stanford University, 2011)
10. Jameson, A., P. Vincent, and P. Castonguay (2012). On the non-linear stability of flux reconstruction schemes. J. Sci. Comput., vol. 50(2)

# The Flux Reconstruction Scheme

The solution is locally represented by Lagrange polynomial of degree $n-1$ on the solution points:

$$u_h = \sum_{j=1}^{n} u_j l_j(x) \qquad f_h^D = \sum_{j=1}^{n} f_j^D l_j(x)$$

The flux is discontinuous and needs to be corrected in a suitable way.

$$\Delta_L = \tilde{f}_L - f_h^D(-1) \qquad \Delta_R = \tilde{f}_R - f_h^D(1)$$

$$g_L(-1) = 1,\ g_L(1) = 0 \qquad g_R(1) = 1,\ g_R(-1) = 0$$

The continuous flux is obtained from the discontinuous counterpart by adding the correction functions of degree $n$ weighted by the flux corrections.

$$f_h^C = \sum_{j=1}^{n} f_j^D l_j(x) + g_L(x)\Delta_L + g_R(x)\Delta_R$$

The continuous flux is finally differentiated at the solution points and the solution is advanced in time.

$$\frac{\partial u_i}{\partial t} + \left[\sum_{j=1}^{n} f_j^D \frac{\mathrm{d}l_j}{\mathrm{d}x}(x_i) + \Delta_L \frac{\mathrm{d}g_L}{\mathrm{d}x}(x_i) + \Delta_R \frac{\mathrm{d}g_R}{\mathrm{d}x}(x_i)\right] = 0$$

**Huynh (2007), AIAA Paper 2007-4079; Huynh (2009), AIAA Paper 2009-403**

# Energy Stability of the FR Scheme

The FR method defines a family of energy stable schemes in the norm.

$$\left\| U^{\delta D} \right\|_{p,2} = \left[ \sum_{n=1}^{N} \int_{x_n}^{x_{n+1}} \left( U_n^{\delta D} \right)^2 + \frac{c}{2} (J_n)^{2p} \left( \frac{\partial^p U_n^{\delta D}}{\partial x^p} \right)^2 \mathrm{d}x \right]^{\frac{1}{2}}$$

The schemes have the form

$$\frac{\partial u_i}{\partial t} + \left[ \sum_{j=1}^{n} f_j^D \frac{\mathrm{d}l_j}{\mathrm{d}x}(x_i) + \Delta_L \frac{\mathrm{d}h_L}{\mathrm{d}x}(x_i) + \Delta_R \frac{\mathrm{d}h_R}{\mathrm{d}x}(x_i) \right] = 0$$

where the correction functions in terms of Legendre polynomials are

$$h_L = \frac{(-1)^p}{2} \left[ L_p - \left( \frac{\eta_p(c) L_{p-1} + L_{p+1}}{1 + \eta_p(c)} \right) \right]$$

$$h_R = \frac{(+1)^p}{2} \left[ L_p + \left( \frac{\eta_p(c) L_{p-1} + L_{p+1}}{1 + \eta_p(c)} \right) \right]$$

with a single parameter $c$

$$\eta_p(c) = \frac{c(2p+1)(a_p p!)^2}{2}$$

**Vincent, et al. (2010), J. Sci. Comput., 47(1); Vincent, et al. (2011), J. Comput. Phys., 230(22)**

# A Family of Energy Stable Schemes

Accuracy

**Nodal DG:**

$$c = 0 \quad \Rightarrow \quad \eta_p = 0$$

$$g_L = \frac{(-1)^p}{2}[L_p - L_{p-1}], \quad g_R = \frac{(+1)^p}{2}[L_p + L_{p+1}]$$

**Spectral Difference:**

$$c = \frac{2p}{(2p+1)(p+1)(a_p p!)^2} \quad \Rightarrow \quad \eta_p = \frac{p}{p+1}$$

$$g_L = \frac{(-1)^p}{2}(1-x)L_p, \quad g_R = \frac{(+1)^p}{2}(1+x)L_p$$

**G2 Scheme by Huynh [2007]:**

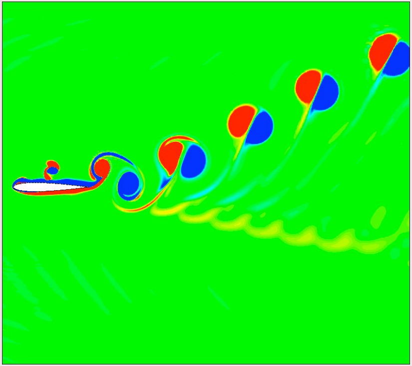
$$c = \frac{2(p+1)}{(2p+1)p(a_p p!)^2} \quad \Rightarrow \quad \eta_p = \frac{p+1}{p}$$

$$g_L = \frac{(-1)^p}{2}\left[L_p - \frac{(p+1)L_{p-1} + pL_{p+1}}{2p+1}\right],$$

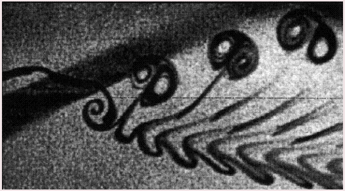
$$g_R = \frac{(+1)^p}{2}\left[L_p + \frac{(p+1)L_{p-1} + pL_{p+1}}{2p+1}\right]$$

Allowable Time-Step

# Study of Flapping Wing Sections



SD, 2D, N=5 on deforming grid



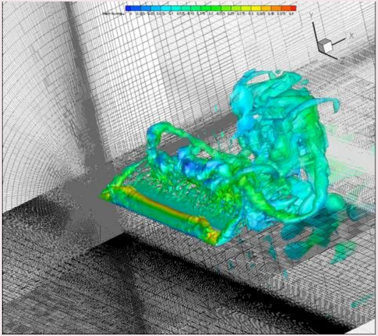
Experiment

Jones, et al. (1998), AIAA J., 36(7)

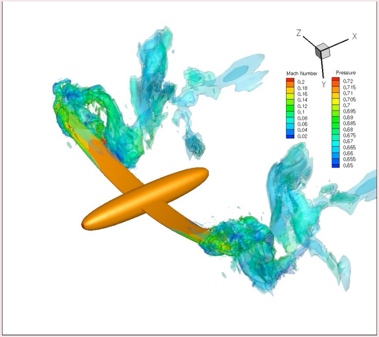
**NACA0012**

$$Re = 1850, Ma = 0.2, St = 1.5,$$
$$\omega = 2.46, h = 0.12c$$

# Flapping Wing Aerodynamics



Iso-Entropy colored by $Ma$
Flapping NACA 0012
$Re = 2000$, SD, $N = 5$, $4.7 \times 10^6$ DoF




Iso-Entropy colored by $Ma$
Wing-Body
$Re = 5000$, SD, $N = 4$, $2.1 \times 10^7$ DoF

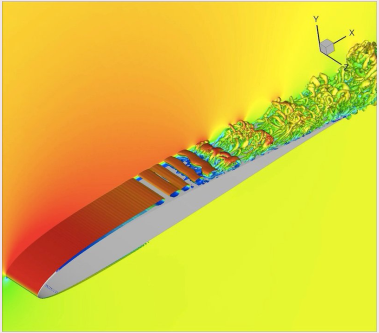
**Ou, et al. (2011), AIAA Paper 2011-1316; Ou and Jameson (2011), AIAA Paper 2011-3068**

# Transitional Flow over SD7003 Airfoil

| | Freestream Turbulence | Separation $x_{sep}/c$ | Transition $x_{tr}/c$ | Reattach. $x_r/c$ |
|---|---|---|---|---|
| Radespiel et al. | 0.08% | 0.30 | 0.53 | 0.64 |
| Ol et al. | 0.10% | 0.18 | 0.47 | 0.58 |
| Galbraith Visbal | 0% | 0.23 | 0.55 | 0.65 |
| Uranga et al. | 0% | 0.23 | 0.51 | 0.60 |
| Present ILES* | **0%** | **0.23** | **0.53** | **0.64** |

Experiments in green

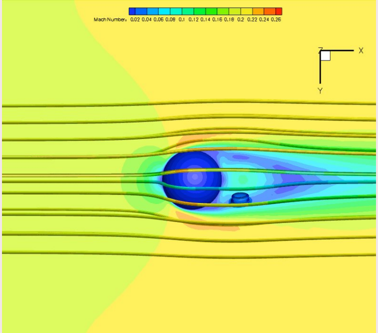
***1.7×10[6] DoF**



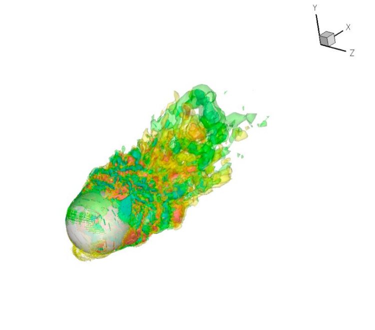

Iso-Entropy colored by $Ma$
Wing-Body
$Re = 5000$, SD, $N = 4$, $2.1 \times 10^7$ DoF

**Castonguay, et al. (2010), AIAA Paper 2010-4626; Radespiel, et al. (2007), AIAA J., 45(6); Ol, et al. (2005), AIAA Paper 2005-5149; Galbraith, Visbal (2008), AIAA Paper 2008-225; Uranga, et al. (2009), AIAA Paper 2009-4131;**

# Flow Over Spheres



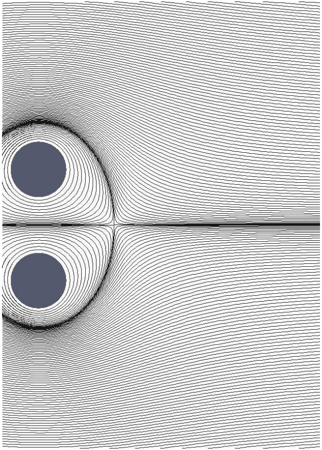

Mach Contours + Streamlines
Flow over a spinning sphere,
$Re = 300,\ Ma = 0.2$



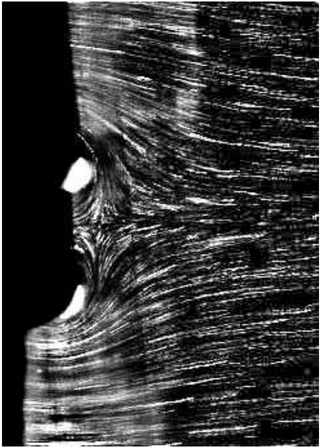

Iso-Entropy colored by $Ma$
Flow over a sphere
$Re = 10000,\ Ma = 0.2$

**Ou, et al. (2011), AIAA Paper 2011-3668**

# Flow Past Counter-Rotating Cylinder Pair: $Re_D = 150, \omega = 3.1\Omega$



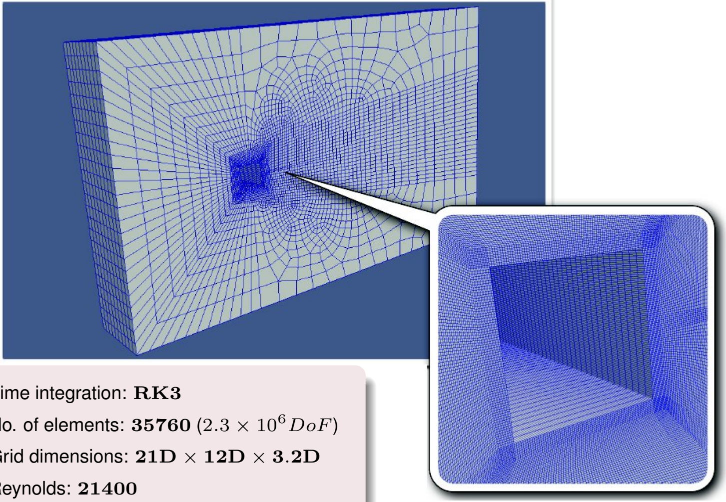
SD, 4th Order, $Ma = 0.2$



Experiment (Princeton)

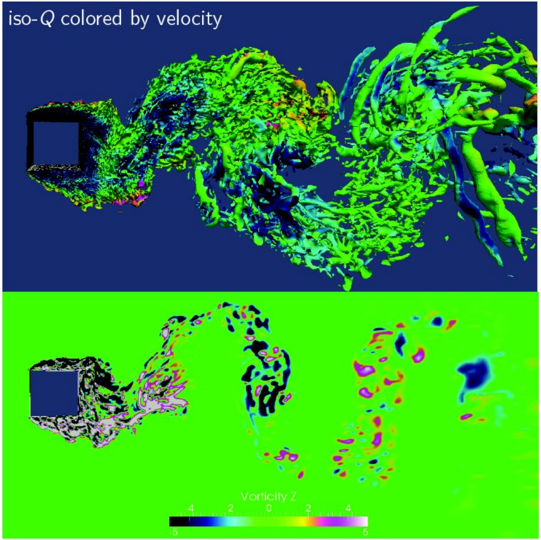
Chan et al. (2011), J. Fluid Mech., Vol. 679, pp. 343-382

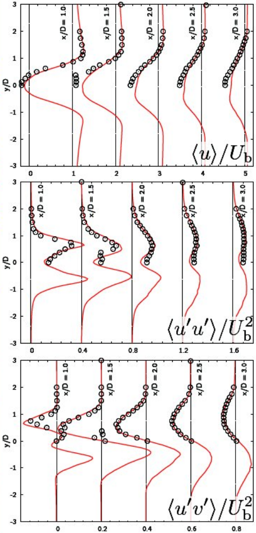
# Large Eddy Simulation of Flow Past Square Cylinder: $Re_D = 21400$



- Time integration: $\mathbf{RK3}$
- No. of elements: $\mathbf{35760}$ $(2.3 \times 10^6\, DoF)$
- Grid dimensions: $\mathbf{21D} \times \mathbf{12D} \times \mathbf{3.2D}$
- Reynolds: $\mathbf{21400}$
- Mach: $\mathbf{0.3}$
- Statistics: $\mathbf{16T_0}$

# Large Eddy Simulation of Flow Past Square Cylinder: $Re_D = 21400$




Preliminary results (work in progress)

# Summary and Conclusions

Predicting the future is generally ill advised.
However, the following are the author's opinions:

- The early development of CFD in the Aerospace Industry was primarily driven by the need to calculate steady transonic flows: **this problem is quite well solved**.
- CFD has been on a plateau for the last 15 years with 2nd-order accurate FV methods for the RANS equations almost universally used in both commercial and government codes which can treat complex configurations.
- These methods cannot reliably predict complex separated, unsteady and vortex dominated flows.
- Ongoing advances in both numerical algorithms and computer hardware and software should enable an advance to LES for industrial applications within the foreseeable future.
- Research should focus on high-order methods with minimal numerical dissipation for unstructured meshes to enable the treatment of complex configurations.
- Eventually DNS may become feasible for high Reynolds number flows...

**hopefully with a smaller power requirement than a wind tunnel.**

# Acknowledgments


## The current research is a combined effort by

- Postdocs: Charlie Liang, Peter Vincent, Guido Lodato
- Ph.D. students: Sachin Premasuthan, Kui Ou, Patrice Castonguay, David Williams, Yves Alleneau, Lala Li, Manuel Lopez and Andre Chan

## It is made possible by the support of

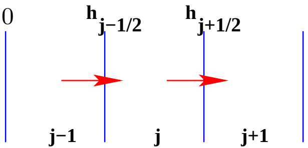
- The Airforce Office of Scientific Research under grant FA9550-10-1-0418 by Dr. Fariba Fahroo
- The National Science Foundation under grants 0708071 and 0915006 monitored by Dr. Leland Jameson

# Appendix

# Artificial Diffusion and LED Schemes

Suppose that the scalar conservation law

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x} f(v) = 0$$




is approximated by the semi-discrete scheme

$$\Delta x \frac{dv_j}{dt} + h_{j+\frac{1}{2}} - h_{j-\frac{1}{2}} = 0$$

where the numerical flux is

$$h_{j+\frac{1}{2}} = \frac{1}{2}\left(f_{j+1} + f_j\right) - \alpha_{j+\frac{1}{2}}\left(v_{j+1} - v_j\right)$$

Define a numerical estimate of the wave speed $a(v) = \frac{\partial f}{\partial v}$ as

$$a_{j+\frac{1}{2}} = \begin{cases} \frac{f_{j+1}-f_j}{v_{j+1}-v_j}, & v_{j+1} \neq v_j \\ \frac{\partial f}{\partial v}\Big|_{v_j}, & v_{j+1} = v_j \end{cases}$$

# Artificial Diffusion and LED Schemes (continued)

then the numerical flux

$$\begin{aligned} h_{j+\frac{1}{2}} &= f_j + \frac{1}{2}\left(f_{j+1} - f_j\right) - \alpha_{j+\frac{1}{2}}\left(v_{j+1} - v_j\right) \\ &= f_j - \left(\alpha_{j+\frac{1}{2}} - \frac{1}{2}a_{j+\frac{1}{2}}\right)\left(v_{j+1} - v_j\right) \end{aligned}$$

and

$$\begin{aligned} h_{j-\frac{1}{2}} &= f_j - \frac{1}{2}\left(f_j - f_{j-1}\right) - \alpha_{j-\frac{1}{2}}\left(v_j - v_{j-1}\right) \\ &= f_j - \left(\alpha_{j-\frac{1}{2}} + \frac{1}{2}a_{j-\frac{1}{2}}\right)\left(v_j - v_{j-1}\right) \end{aligned}$$

The semi-discrete scheme then reduces to

$\Delta x \frac{dv_j}{dt} = (\alpha_{j+\frac{1}{2}} - \frac{1}{2}a_{j+\frac{1}{2}})(v_{j+1} - v_j) - (\alpha_{j-\frac{1}{2}} + \frac{1}{2}a_{j-\frac{1}{2}})(v_j - v_{j-1})$

This is LED if $\alpha_{j+\frac{1}{2}} \geq \frac{1}{2}|a_{j+\frac{1}{2}}| \quad \forall j$.

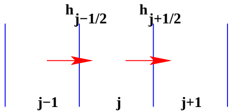
# Jameson-Schmidt-Turkel (JST) Scheme

This scheme blends low and high order diffusion.
Suppose that the scalar conservation law

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x} f(v) = 0$$

is approximated by the semi-discrete scheme

$$\Delta x \frac{dv_j}{dt} + h_{j+\frac{1}{2}} - h_{j-\frac{1}{2}} = 0$$




In the JST scheme, the numerical flux is

$$h_{j+\frac{1}{2}} = \frac{1}{2}(f_{j+1} + f_j) - d_{j+\frac{1}{2}}$$

where the diffusive flux has the form

$$d_{j+\frac{1}{2}} = \epsilon^{(2)}_{j+\frac{1}{2}} \Delta v_{j+\frac{1}{2}} - \epsilon^{(4)}_{j+\frac{1}{2}} (\Delta v_{j+\frac{3}{2}} - 2\Delta v_{j+\frac{1}{2}} + \Delta v_{j-\frac{1}{2}})$$

with

$$\Delta v_{j+\frac{1}{2}} = v_{j+1} - v_j$$

# JST Scheme

Let $a_{j+\frac{1}{2}}$ be an estimate of the wave speed $\frac{\partial f}{\partial v}$

$$a_{j+\frac{1}{2}} = \frac{f_{j+1} - f_j}{v_{j+1} - v_j}$$

or

$$\frac{\partial f}{\partial v}\Big|_{v=v_j} \quad \text{if} \quad v_{j+1} = v_j$$

**Theorem**: The JST scheme is LED if whenever $v_j$ or $v_{j+1}$ is an extremum

$$\epsilon^{(2)}_{j+\frac{1}{2}} \geq \frac{1}{2}|a_{j+\frac{1}{2}}|, \quad \epsilon^{(4)}_{j+\frac{1}{2}} = 0$$

**Proof**: At an extremum the scheme reduces to

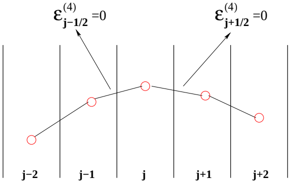
$$\Delta_x \frac{dv_j}{dt} = \left(\epsilon^{(2)}_{j+\frac{1}{2}} - \frac{1}{2}a_{j+\frac{1}{2}}\right)\Delta v_{j+\frac{1}{2}} - \left(\epsilon^{(2)}_{j-\frac{1}{2}} + \frac{1}{2}a_{j-\frac{1}{2}}\right)\Delta v_{j-\frac{1}{2}}$$

where each term in parenthesis $\geq 0$.

# JST Scheme at a Maximum

The condition that $\epsilon^{(4)}_{j+\frac{1}{2}} = 0$ if $v_j$ or $v_{j+1}$ is an extremum

$$\Longrightarrow \epsilon^{(4)}_{j+\frac{1}{2}} = \epsilon^{(4)}_{j-\frac{1}{2}} = 0.$$




Hence the scheme reduces to a 3-point scheme and

$$\frac{dv_j}{dt} \leq 0$$

if

$$\epsilon^{(2)}_{j+\frac{1}{2}} \geq \frac{1}{2}|a_{j+\frac{1}{2}}|, \quad \epsilon^{(2)}_{j-\frac{1}{2}} \geq \frac{1}{2}|a_{j-\frac{1}{2}}|,$$

since then the coefficients multiplying $(v_{j+1} - v_j)$ and $(v_{j-1} - v_j)$ are both $\geq$ 0.

# Switch for JST Scheme

Define

$$R(u,v) = \left| \frac{u-v}{|u|+|v|} \right|^q, \quad q \geq 1$$

Set

$$Q_{j+\frac{1}{2}} = R(\Delta v_{j+\frac{3}{2}}, \Delta v_{j-\frac{1}{2}})$$

$$\epsilon^{(2)}_{j+\frac{1}{2}} = \alpha_{j+\frac{1}{2}} Q_{j+\frac{1}{2}}$$

$$\epsilon^{(4)}_{j+\frac{1}{2}} = \beta_{j+\frac{1}{2}} (1 - Q_{j+\frac{1}{2}})$$

Then the scheme is LED if

$$\alpha_{j+\frac{1}{2}} \geq \frac{1}{2} |a_{j+\frac{1}{2}}|.$$

If

$$\beta_{j+\frac{1}{2}} = \frac{1}{2} \alpha_{j+\frac{1}{2}},$$

the JST scheme reduces to the SLIP scheme.