Past or Future?

A Never-Ending Story of CFD Algorithm Development

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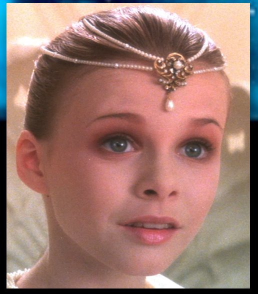
Four Decades of CFD: Looking Back and Moving Forward:
A Symposium Celebrating the Careers of Antony Jameson, Phil Roe and Bram van Leer

June 22-23, 2013

Click the paper number to download the pdf.

THE NEVERENDING STORY

How many wishes do I get?



As many as you want. And the more wishes you make, the more magnificent CFD will become.

My Michigan Years

1994 - 2007

Courses: Aerodynamics II, CFD I (**Roe**)
Compressible Flows, CFD II (**Van Leer**)

Research: Adaptive grids and solutions (PhD)
Multigrid
Theory of local preconditioning
Multidimensional upwind schemes
Elliptic/Hyperbolic Splitting
Carbuncle, rotate-hybrid Riemann solvers
High-order methods....

I'm just one of many Michigan CFDers.

Michigan CFDers

Bob Biedron
Nelson Carter
Necdet Aslan
Karim Mazaheri
David W. Levy
Daren De Zeeuw
William Coirier
Wen-Tzong Lee
Chris Rumsey
Andrew Cary
Lisa Mesaros
Timothy G. Tomaich
John Lynn
Shawn Brown
Rob Lowrie
Mohit Arora
Gregory Ashford
Dohyung Lee

Jens-Dominik Mueller
Clinton Groth
Dave Darmofal
Sami Byyuk
Grenmarie Agresar
Jeffrey Thomas
Rho Shin Myong
Brian Nguyen
Eric Charlton
Cheolwan Kim
Dawn Kinsey
Timur Linde
Jeff Benko
David Mott
Jason Hunt
Shuichi Nakazawa
Jeffrey Hittinger
Constantin Kabin

Christian Aalburg
Mani Rad
Hiro Nishikawa
Bil Kleb
Greg Burton
Teppei Hojo
Chad Ohlandt
Chris Depcik
Farzad Ismail
Keiichi Kitamura
Paul Kominsky
Yoshifumi Suzuki
Marcus Lo
Loc Khieu
Paul Ullrich
Daniel Zaide
Tim Eymann
Kaihua Ding
Marco Ceze

Let me know if any name is missing!

Techniques for Hyperbolic Systems

Great progress made in past decades for hyperbolic systems.

1. MUSCL scheme (kappa scheme, limiters)
2. Upwind fluxes (Riemann solvers, FVS)
3. Stiff-relaxation schemes
4. Multidimensional upwinding
5. Local Preconditioning
6. Elliptic/Hyperbolic splitting
7. Entropy-consistent/stable flux

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Tremendous influence on modern numerical methods.

High-Resolution Schemes

Barrier breaking: Monotone 2nd-order upwind schemes

Van Leer's kappa scheme 1985 (non-limited version)

$$
\begin{aligned}
u_L &= \overline{u}_j + \frac{1}{4}\left[(1+\kappa)(\overline{u}_{j+1} - \overline{u}_j) + (1-\kappa)(\overline{u}_j - \overline{u}_{j-1})\right] \\
u_R &= \overline{u}_j - \frac{1}{4}\left[(1+\kappa)(\overline{u}_{j+1} - \overline{u}_j) + (1-\kappa)(\overline{u}_{j+2} - \overline{u}_{j+1})\right]
\end{aligned}
$$

kappa=1/3 recovers the quadratic reconstruction of *point-values* from *cell-averages*, leading to 3rd-order accurate advection scheme.

Kappa Scheme is employed in practical CFD codes:
NASA's CFL3D, FUN3D, Software Cradle's SC/Tetra, etc.

Upwind Fluxes

Robust and accurate Euler fluxes

Roe's Approximate Riemann Solver (1981)
Van Leer's Flux-Vector Splitting(1982) and HLL flux (1983)

Default in modern practical CFD solvers:

- NASA's FUN3D: Jac(Van Leer) $\Delta \mathrm{U}$ = RHS(Roe)
- Software Cradle's SC/Tetra: Rotated-Roe-HLL flux (Nishikawa&Kitamura JCP2007)

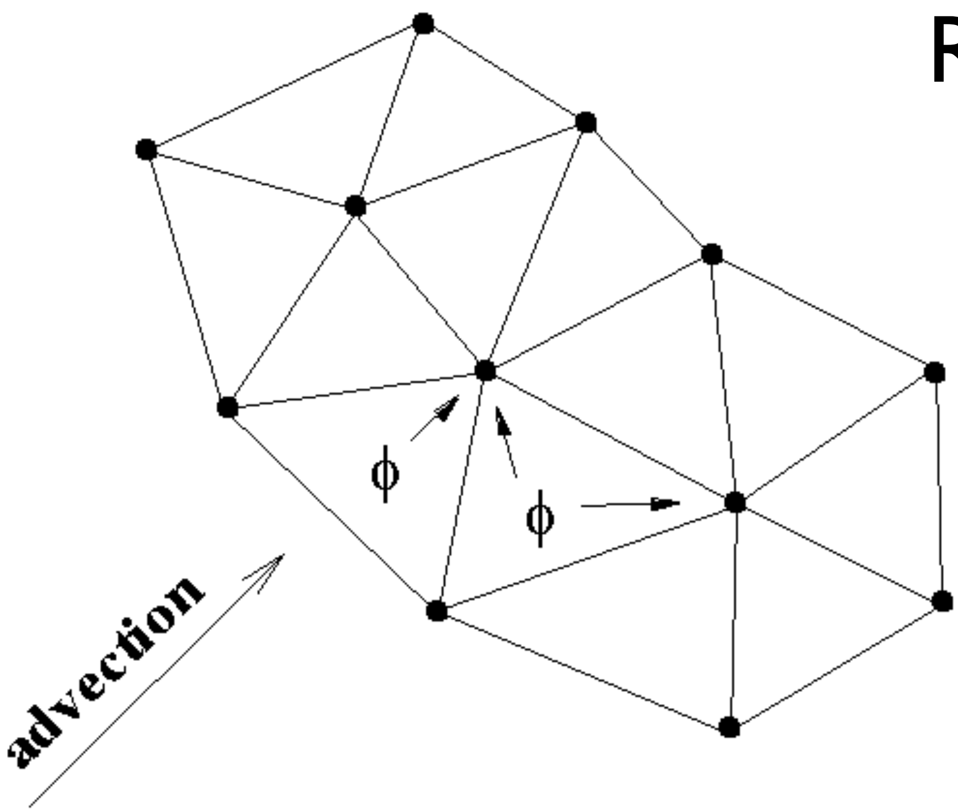
Many other useful fluxes generated in similar approaches

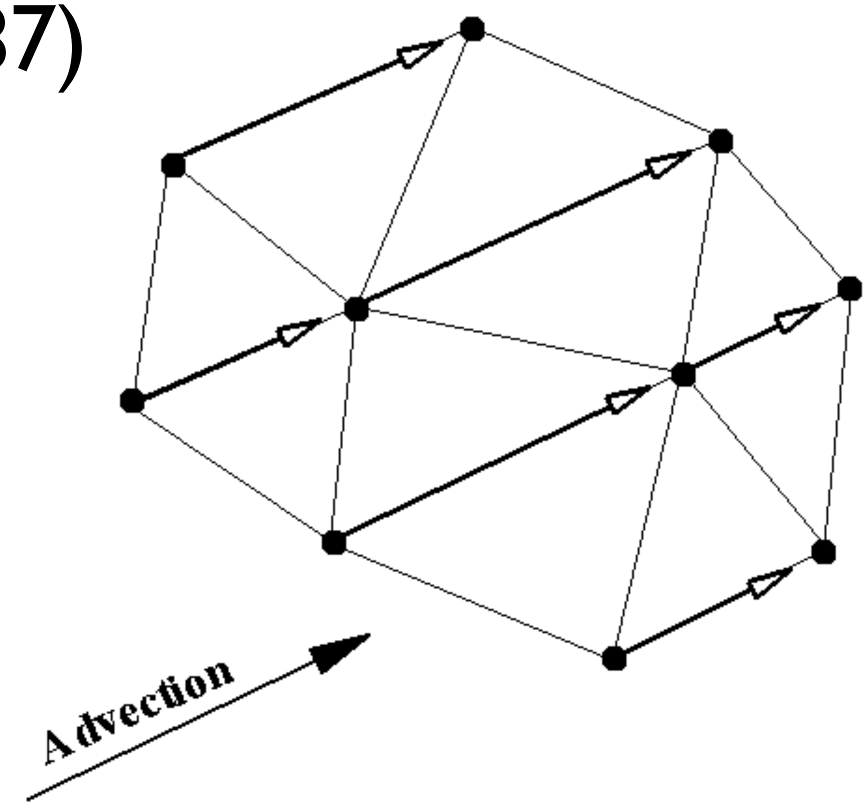
Multidimensional Upwind

Beyond 1D Physics

Fluctuation-Splitting/Residual-Distribution

Roe (1987)





Europe (VKI, INRIA, etc.), University of Michigan, NASA Langley:

Low-diffusion, compact, monotone high-order, optimal discretization by decomposition, adaptive quadrature (IJNMF2008), adaptive grids(IJNMF2002), Petrov-Galerkin, etc.

Very active area of CFD algorithm research

Local Preconditioning

Two for the price of one

Convergence acceleration and low-Mach accuracy recovery

Artificial Compressibility (Chorin)
Systematic development/applications (Turkel, Merkle, etc.)

Van-Leer-Lee-Roe Preconditioner (1991)

- *Elliptic/Hyperbolic splitting*
  * Optimal multigrid convergenve (full/semi-coarsening) JCP2002
  * Optimal discretization (isotropic, upwind) AIAA1995
    Third-order Euler code based on decomposition, AIAA2001
  * A general construction method for 2D PDEs, MHD in AIAA2003

Local-preconditioning employed in many practical CFD solvers

Future: Moving Forward

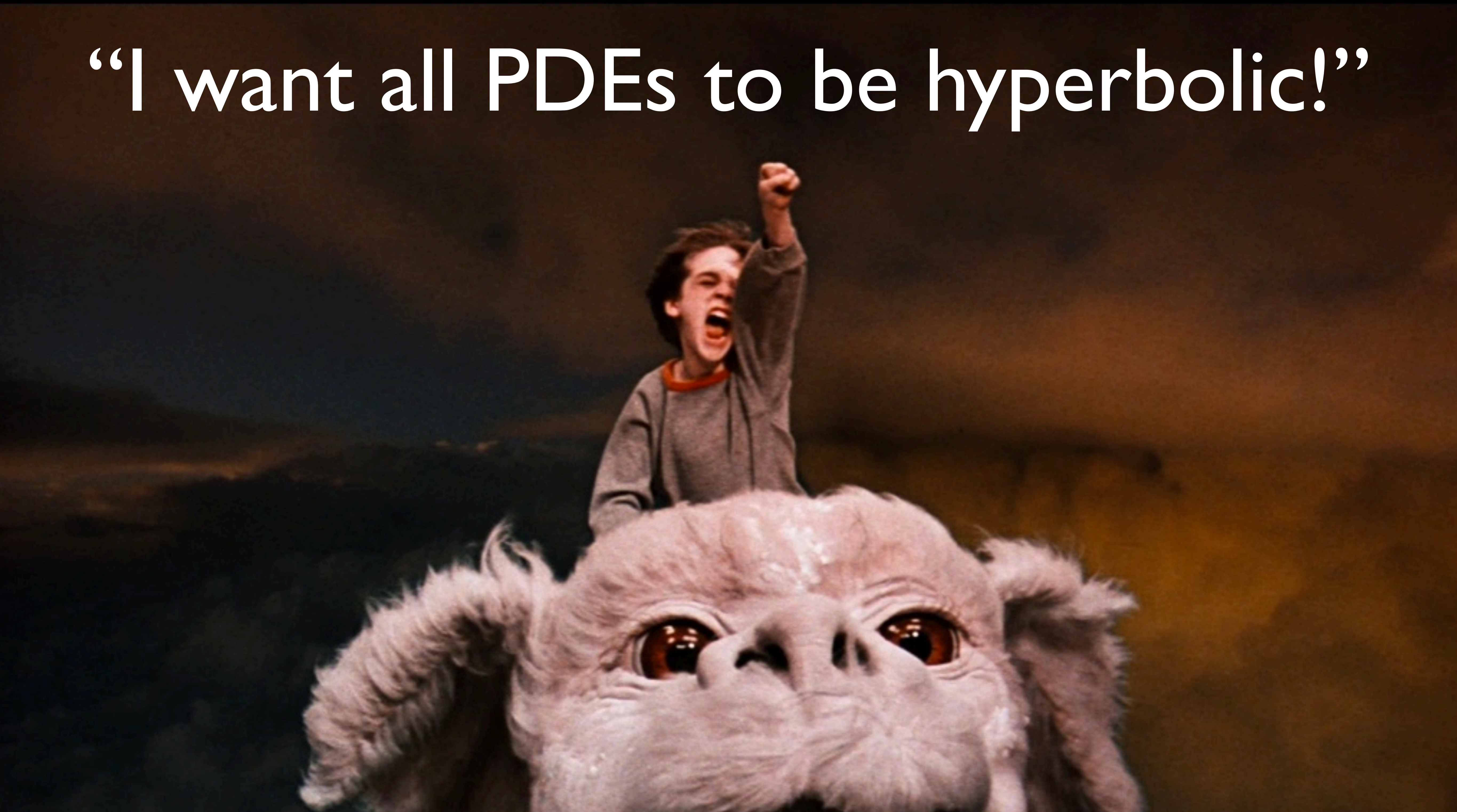
Beyond Hyperbolic Systems

Future?

I’ve just talked about it.

“What?????”   Well, it is time to tell you about the wish I made.

“I want all PDEs to be hyperbolic!”



A new CFD world arises from your dreams and wishes.

Hyperbolize Them All

JCP2007, 2010, 2012 (source term), AIAA2009, 2010tr, 2011, 2011tr, 2013d, 2013ad, CF2011tr

$$\mathbf{U}_t + \mathbf{A}\mathbf{U}_x = \mathbf{B}\mathbf{U}_{xx} + \mathbf{C}\mathbf{U}_{xxx} + \cdots + \mathbf{S}$$

$$\Downarrow$$

$$\tilde{\mathbf{W}}_t + \tilde{\mathbf{A}}\tilde{\mathbf{W}}_x = 0$$

Dramatic simplification/improvements to numerical methods

Methods for hyperbolic systems directly apply to all Partial Differential Equations.

Note: 2010tr, CF2011tr, and 2011tr are about the construction of traditional diffusion/viscous schemes via the hyperbolic system method. Warning: AIAA 2010-5093 is an 82-page AIAA paper

Burgerize Them All

Simple, Efficient, Accurate.

Looks eccentric? But the taste is the same, or even better!

Nishikawa, JCP2007, AIAA2013

Hyperbolic Diffusion System

Not the mixed form nor stiff-relaxation systems

$$\partial_t u = \nu \left(\partial_{xx} u + \partial_{yy} u\right)$$

Sushi Burger for Diffusion

steady

$$\begin{aligned} \partial_t u &= \nu\left(\partial_x p + \partial_y q\right) \\ \partial_t p &= \left(\partial_x u - p\right)/T_r \\ \partial_t q &= \left(\partial_y u - q\right)/T_r \end{aligned} \quad \rightarrow \quad \begin{aligned} 0 &= \nu\left(\partial_x p + \partial_y q\right), \\ p &= \partial_x u, \\ q &= \partial_y u, \end{aligned} \quad \rightarrow \quad 0 = \nu\left(\partial_{xx} u + \partial_{yy} u\right),$$

System is equivalent to diffusion in the steady state for *any Tr* :

$$T_r = \frac{L_r^2}{\nu}, \quad L_r = \frac{1}{2\pi}$$

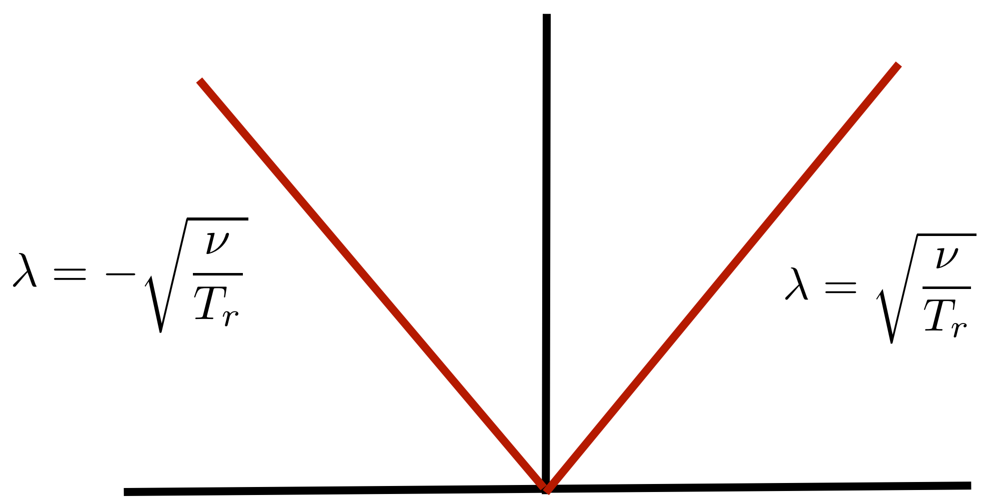
Optimized for fast convergence (JCP2007, JCP2010 , AIAA2013)

Unsteady computation possible by implicit time-stepping with a steady solver used in the inner iteration. (Alireza Mazaheri, NASA LaRC)

Upwind Scheme for Diffusion

$$\mathbf{U}_t + \mathbf{A}\mathbf{U}_x = \mathbf{Q}, \quad \mathbf{U} = \begin{bmatrix} u \\ p \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & -\nu \\ -1/T_r & 0 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} 0 \\ -p/T_r \end{bmatrix}$$

Waves travel isotropically:



(Note: Elliptic in space)

E.g., Upwind scheme

$$\mathbf{F}_{j+1/2} = \frac{1}{2}\left[\mathbf{F}_{j+1} + \mathbf{F}_j\right] - \frac{1}{2}|\mathbf{A}|\left(\mathbf{U}_{j+1} - \mathbf{U}_j\right)$$

Upwinding results in a symmetric stencil due to the symmetric wave structure.

Rapid steady convergence with O(h) time step,
Solution and gradient to the same order of accuracy.

Nishikawa, AIAA 2011-3043

Hyperbolic Navier-Stokes System

Traditional NS System

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0,$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + p - \tau)}{\partial x} = 0,$$

$$\frac{\partial(\rho E)}{\partial t} + \frac{\partial(\rho u H - \tau u + q)}{\partial x} = 0,$$

$$\tau = \mu_v \frac{\partial u}{\partial x},$$

$$q = -\frac{\mu_h}{\gamma(\gamma-1)}\frac{\partial T}{\partial x}.$$

$$\left(\mu_v = \frac{4}{3}\mu, \quad \mu_h = \frac{\gamma\mu}{Pr}.\right)$$

Hyperbolic NS System

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0,$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + p - \tau)}{\partial x} = 0,$$

$$\frac{\partial(\rho E)}{\partial t} + \frac{\partial(\rho u H - \tau u + q)}{\partial x} = 0,$$

$$\frac{\partial \tau}{\partial t} - \frac{1}{T_v}\left(\mu_v \frac{\partial u}{\partial x} - \tau\right) = 0,$$

$$\frac{\partial q}{\partial t} - \frac{1}{T_h}\left(-\frac{\mu_h}{\gamma(\gamma-1)}\frac{\partial T}{\partial x} - q\right) = 0,$$

$$T_v = L^2/\nu_v, \quad T_h = L^2/\nu_h$$

Two systems are equivalent in the steady state.

Nishikawa, AIAA 2011-3043

Preconditioned Conservative System

$$\mathbf{P}^{-1}\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S}$$

Inviscid

$$\mathbf{P}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & T_v/\mu_v & 0 \\ 0 & 0 & 0 & 0 & T_h/\mu_h \end{bmatrix}, \mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho E \\ \tau \\ q \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p - \tau \\ \rho u H - \tau u + q \\ -u \\ \dfrac{a^2}{\gamma(\gamma-1)} \end{bmatrix}, \mathbf{S} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tau/\mu_v \\ q/\mu_h \end{bmatrix}$$

Inviscid and Viscous Jacobians:

$$\mathbf{PA} = \mathbf{PA}^{\mathbf{i}} + \mathbf{PA}^{\mathbf{v}}, \quad \mathbf{A} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}}.$$

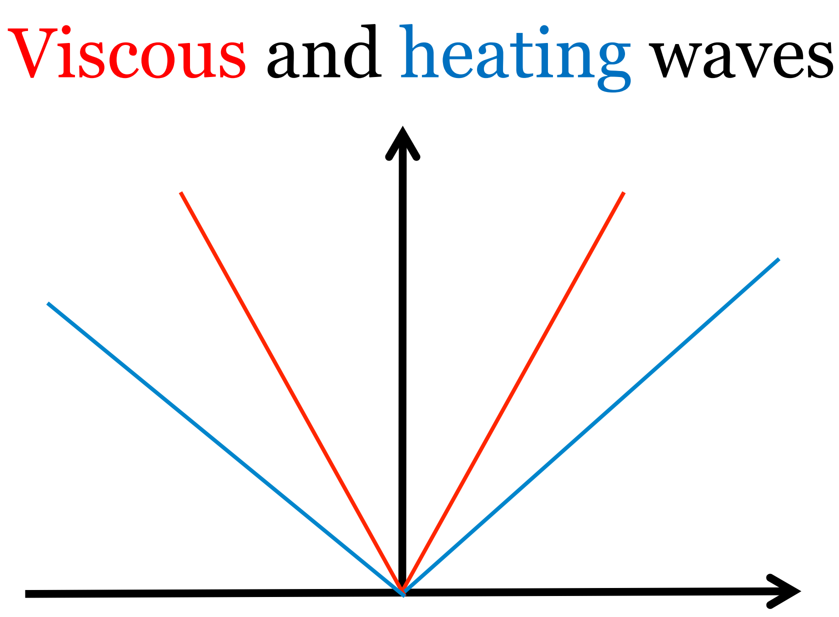
Eigen-structure of each Jacobian is fully analyzable.

Nishikawa, AIAA 2011-3043

Viscous Terms are Hyperbolic

Viscous Jacobian has real eigenvalues:

$$\lambda^v = \pm\sqrt{\frac{\nu_v}{T_v}}, \quad \pm\sqrt{\frac{\nu_h}{T_h}}, \quad 0$$



Navier-Stokes Equations = Hyperbolic Inviscid + Hyperbolic Viscous

All we need are methods for hyperbolic systems.

Methods Already Available for Diffusion

Diffusion

Multidimensional Upwind for Diffusion (JCP2007 JCP2010)

Lax-Wendroff(JCP2007), LDA(JCP2010)

MUSCL and Upwind Flux for Diffusion (AIAA2013)

Viscous Terms (AIAA2011)

MUSCL scheme for the viscous terms

Upwind flux (Riemann solver) for the viscous terms

Local-Preconditioning formulation

Past or Future?

Advantages

1. Discretization made simple and straightforward

- Schemes and techniques developed for advection can be directly applied to diffusion - *multi-dimensional upwind, high-order, etc.*
- 1st-order viscous schemes (robustness, consistent Jacobian, P0 DG)

2. O(1/h) acceleration in convergence (low-Reynolds)

- O(h) time step for explicit schemes
- O(1/h) condition number for linear system in implicit schemes

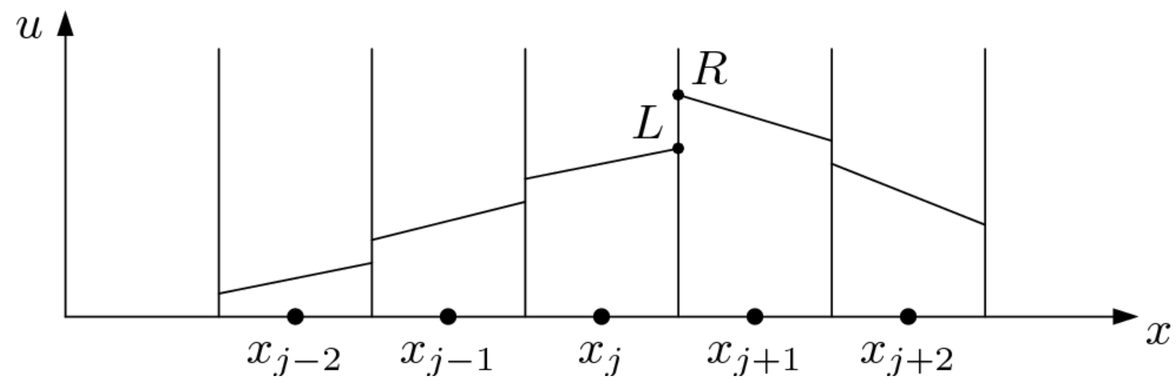
3. Higher-order derivatives (viscous/heat fluxes)

- Same order of accuracy for solution and derivatives

If you have a good inviscid scheme,
you have a <u>very</u> good viscous scheme.

Nishikawa, AIAA 2011-3043

Upwind Flux for Viscous Terms



Finite-volume method:

$$\mathbf{P}_j^{-1}\frac{d\mathbf{U}_j}{dt} = -\frac{1}{\Delta x}\left[\mathbf{F}_{j+1/2} - \mathbf{F}_{j-1/2}\right] + \mathbf{S}_j$$

Upwind Flux for Navier-Stokes:

$$\mathbf{F}_{j+1/2} \;=\; \frac{1}{2}\left[\mathbf{F}_R + \mathbf{F}_L\right] - \frac{1}{2}\mathbf{P}^{-1}\left|\mathbf{PA}\right|\left(\mathbf{U}_R - \mathbf{U}_L\right)$$

where

$$|\mathbf{PA}| \approx |\mathbf{PA}^i| + |\mathbf{PA}^v|$$

All we need are methods for hyperbolic systems.

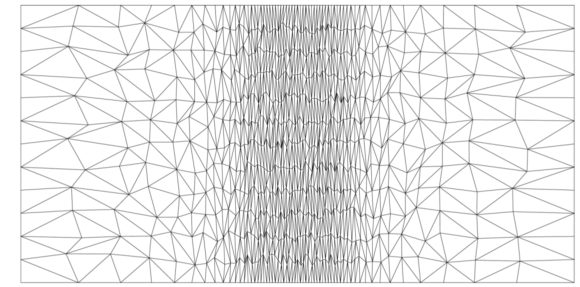
Nishikawa, AIAA 2011-3043

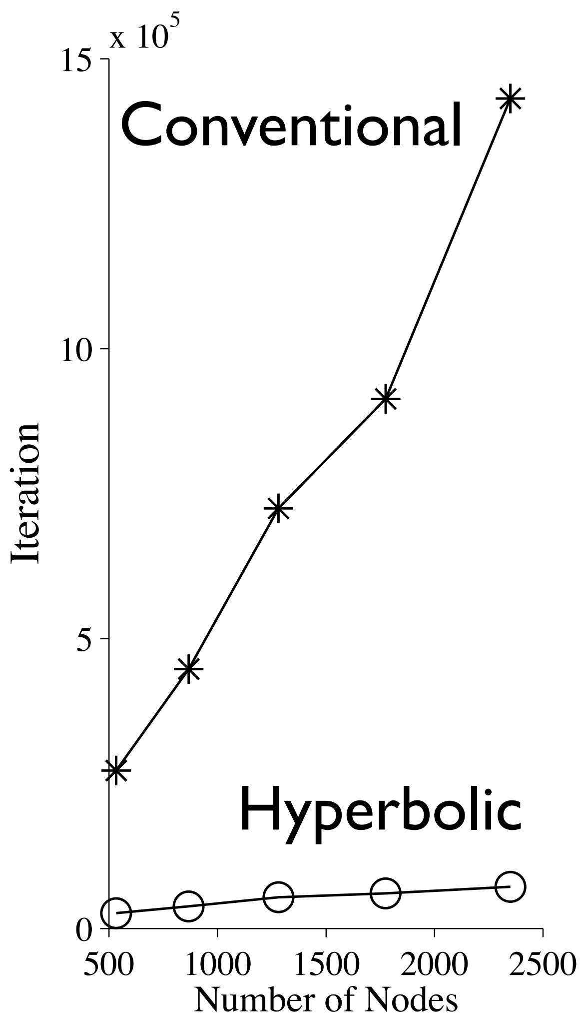
Upwind Navier-Stokes Scheme

2nd-order finite-volume schemes

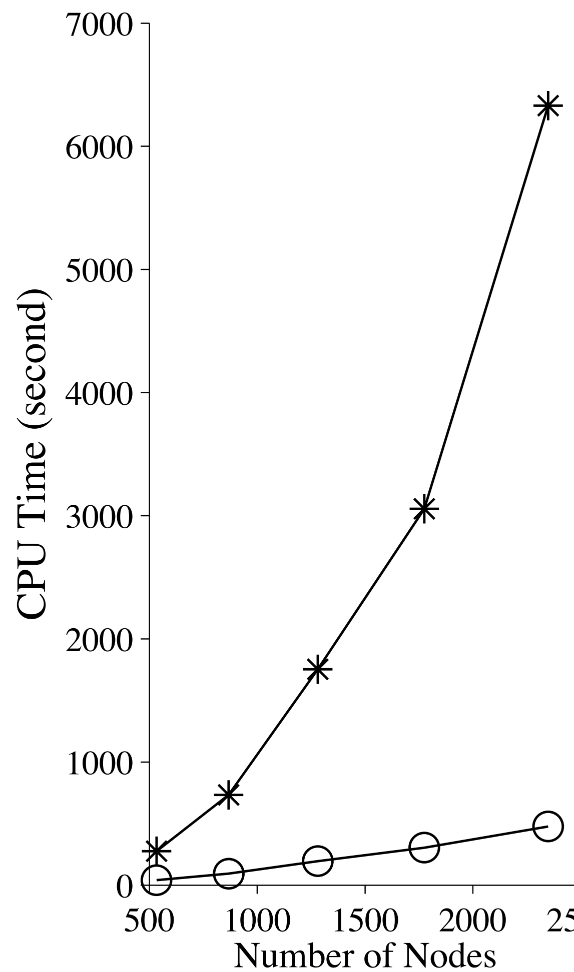
Viscous Shock-Structure Problem

M_∞=3.5, Pr=$\frac{3}{4}$, γ=1.4, Re_∞=25, T_∞=400[K].

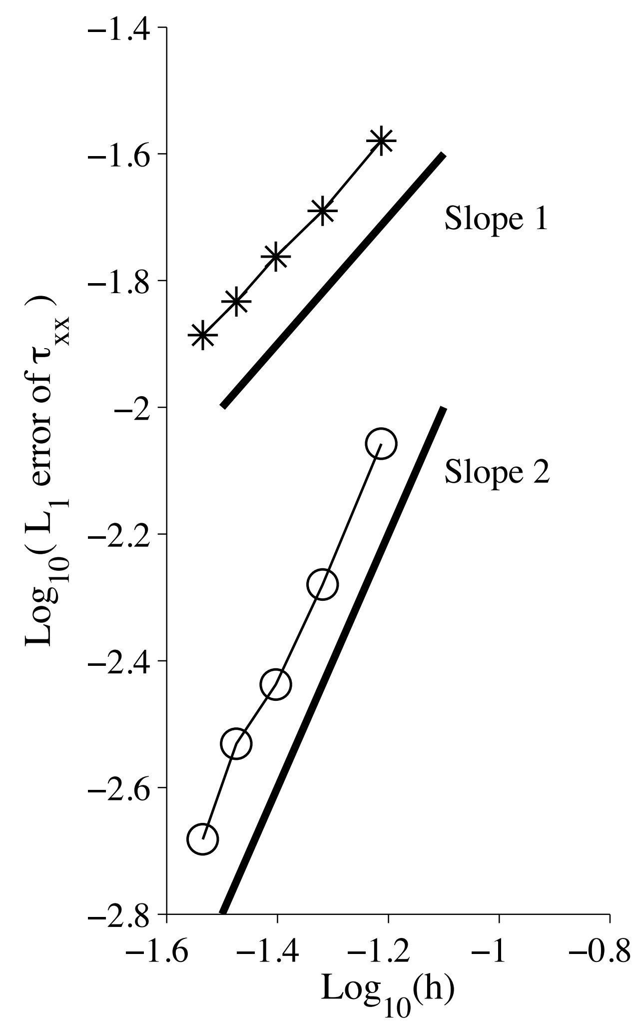




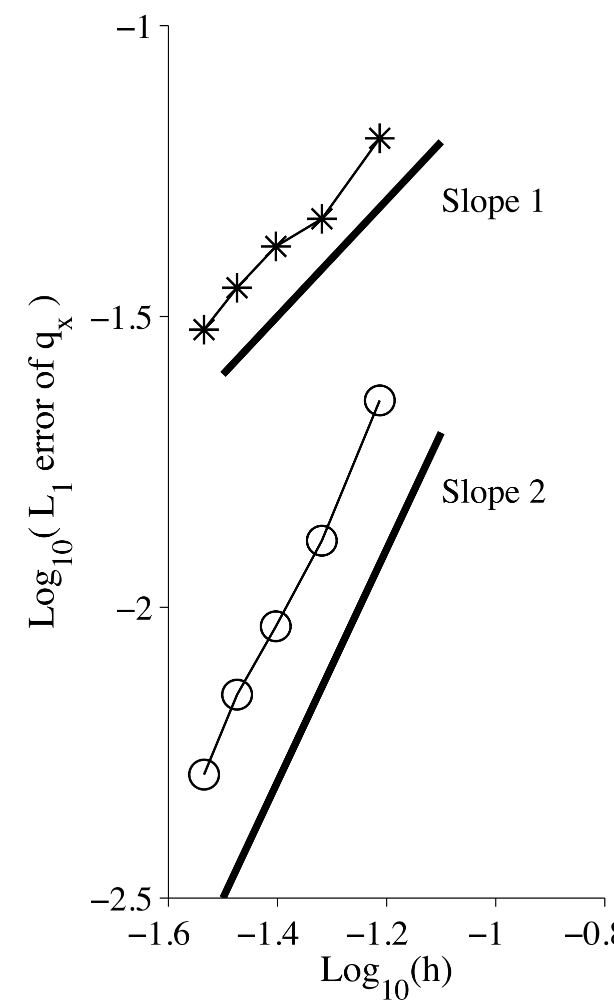
Iteration



CPU Time



Viscous Stress



Heat Flux

Higher-order accuracy with accelerated convergence

Recent Development

Economical high-order finite-volume schemes

Third-order edge-based FV scheme - Katz&Sankaran2011

2nd-order FV with quadratic LSQ gradients (for hyperbolic systems)

2013 1st/2nd/3rd order FV schemes for Diffusion (AIAA 2013-1125)

2013 1st/2nd/3rd order FV schemes for Advection-Diffusion (AIAA 2013-2568)

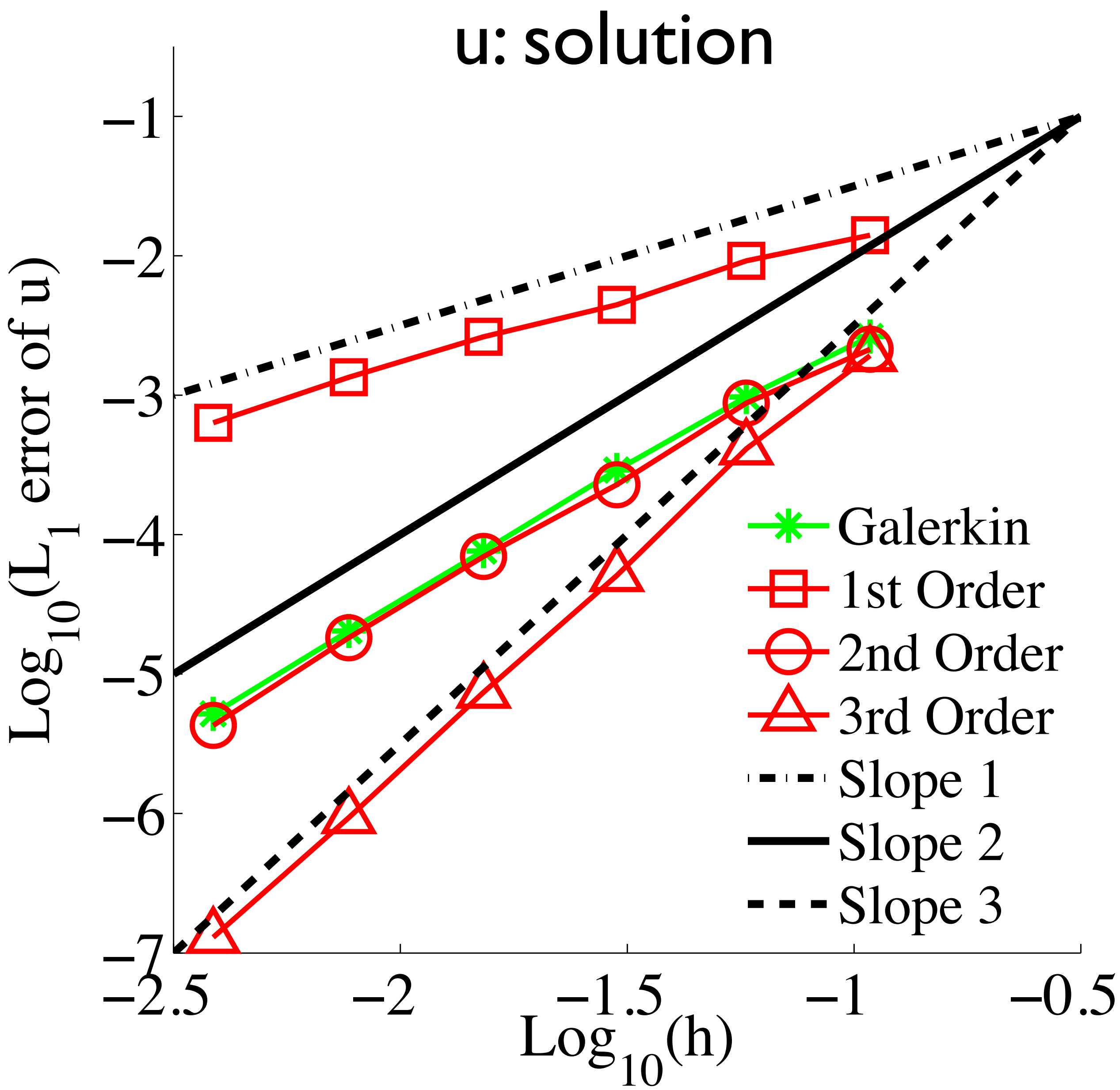
2014 1st, 2nd, 3rd order FV schemes for Navier-Stokes

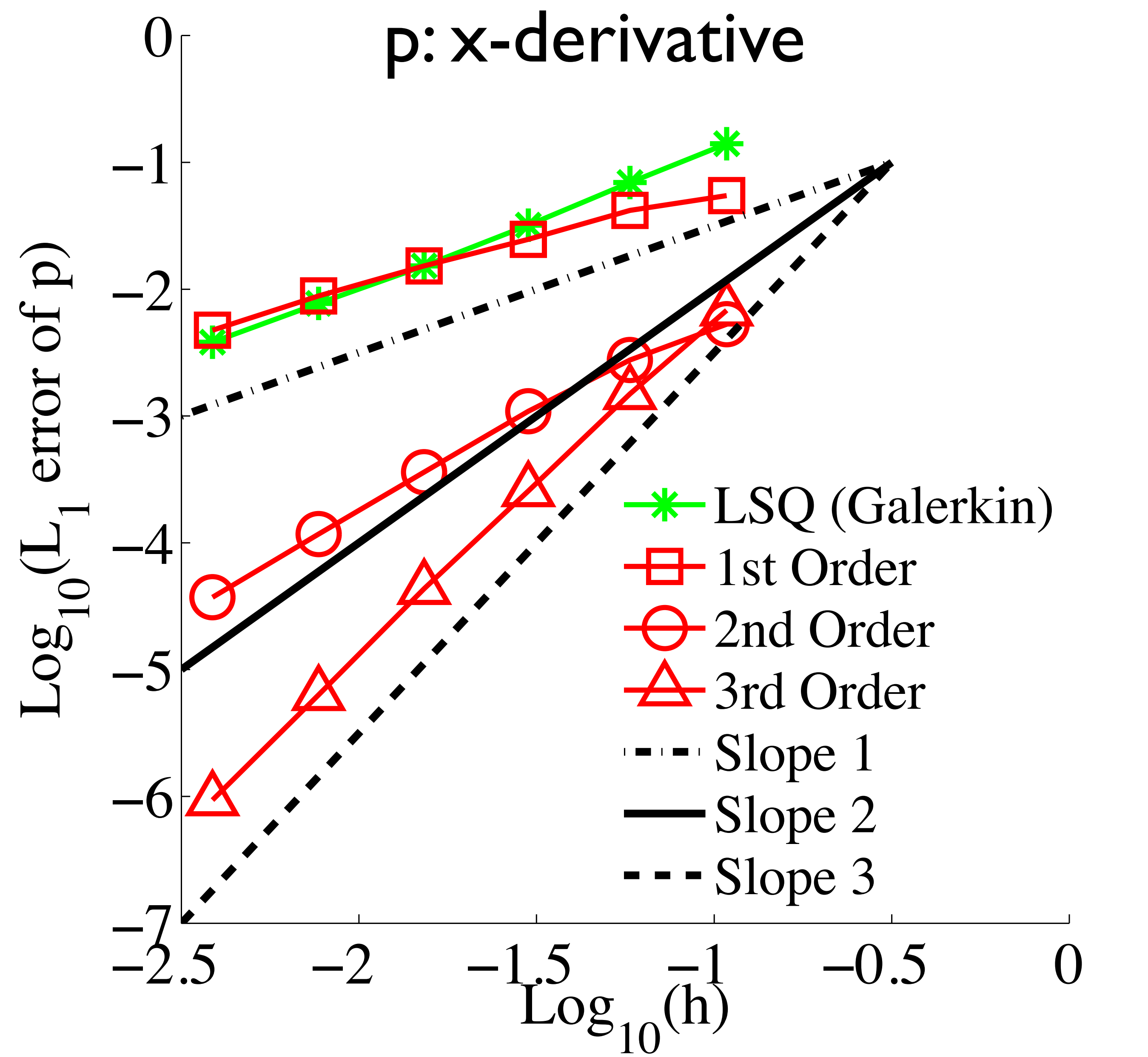
Third-order accuracy for solution and derivatives nearly at the cost of 2nd-order FV scheme.

Nishikawa, AIAA 2013-1125

1st, 2nd, 3rd Order FV Schemes

Diffusion on Unstructured Triangular Grids





3rd-order accurate solution and gradients.

Nishikawa, AIAA 2013-1125

Cost Comparison

Cost per time step

	Galerkin	First-Order	Second-Order	Third-Order
Forward Euler	0.66	1.00	1.26	1.33

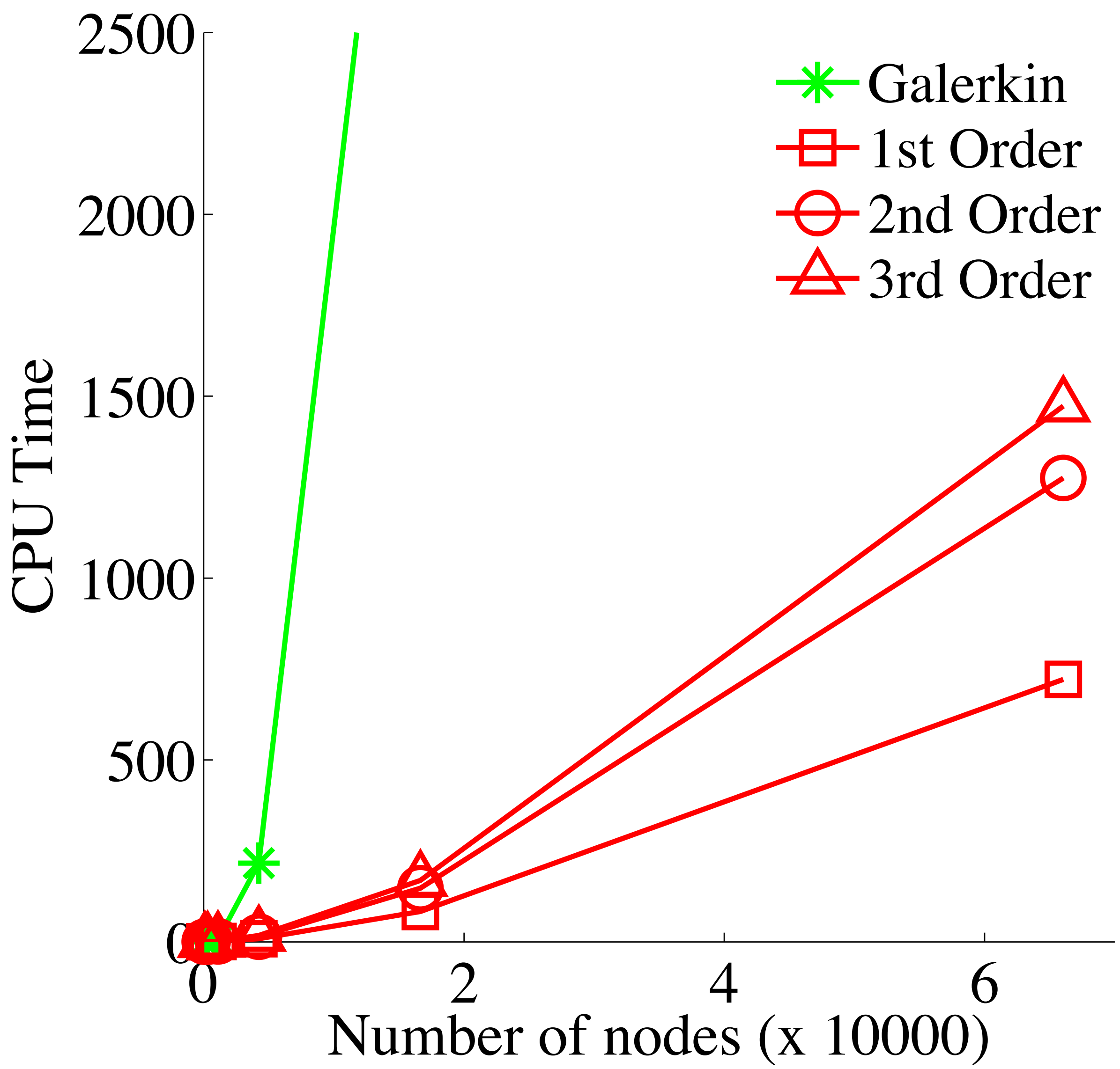
Almost the same.

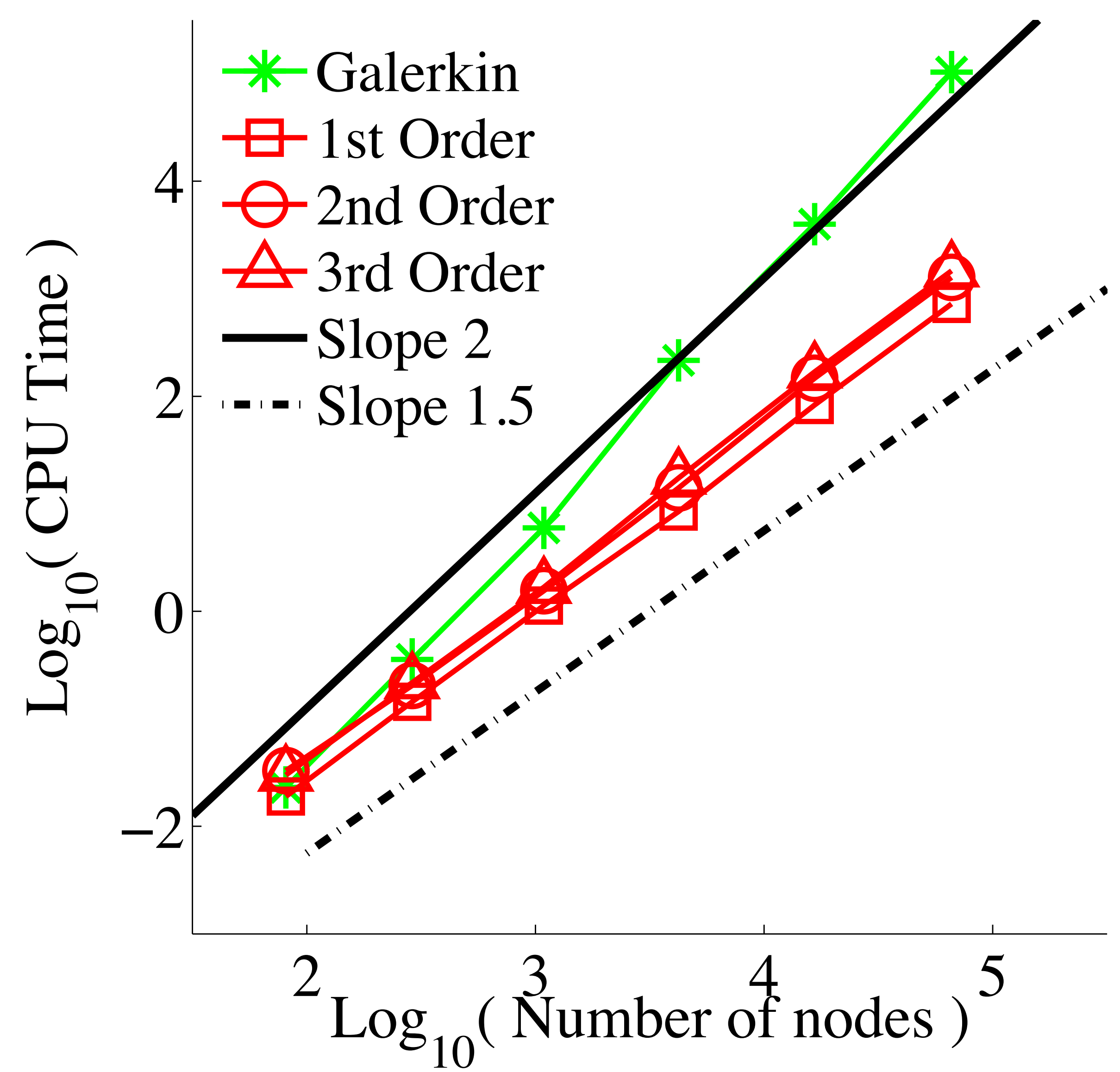
Reality is that hyperbolic schemes are more economical because they converge O(1/h) faster than typical diffusion schemes.

Nishikawa, AIAA 2013-1125

Time to Solution

O(1/h) acceleration overwhelms the increased cost per time step.





Orders of magnitude acceleration in CPU time

NOTE: Speed-up factor grows

Nishikawa, AIAA 2013-2568

1st, 2nd, and 3rd-Order FV Schemes for Advection-Diffusion

Key Features:

1. Upwind for all: Advection, Diffusion, Source Terms
2. 1st, 2nd, and 3rd order accurate solution and gradients
3. Uniform accuracy for all Reynolds numbers
4. Efficient implicit solver with consistent Jacobian
5. Higher-order in advection limit (1st/2nd → 2nd/3rd)

To be presented tomorrow at 2:30pm.

(or NIA CFD Seminar video is available at http://www.hiroakinishikawa.com/niacfds).

Various Opportunities

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I let you enjoy thinking about what we can do if all PDEs are hyperbolic.

Progress Repeats Itself

As PDEs turned into hyperbolic systems:

3rd-derivatives, 4th-derivatives, 5th-derivatives,.....,

Methods for Hyperbolic Systems developed in the past four decades will always be the state-of-the-art and the next generation.

This is what I mean by ...

THE NEVERENDING STORY

of CFD Algorithm Development

I want all PDEs to be hyperbolic !

Wonderful! Look, CFD begins to shine again like it used to, and will keep shining forever.