CFD Before CFD

Bill Rider and Ann Mattsson
June 23, 2013
JRV Symposium
CFD originated from the efforts of many great minds

Lord Rayleigh & G. I. Taylor

Courant, Friedrichs, Lewy – 1928 paper

John Von Neumann

Bethe and Feynman – the first calculations using Von Neumann’s method at Los Alamos in 1944

Peter Lax

Teller, Metropolis, Ulam – Monte Carlo Methods and the H-Bomb

Robert Richtmyer

Harlow – the name CFD and Los Alamos often conjures

Sir Rudolf Peierls

Landshoff & Rosenbluth

Godunov

Landshoff
The first “CFD” calculations

• The first hydro calculation was described in a Los Alamos report (LA-94) on June 20, 1944 – lead author Hans Bethe
  – Feynmann was the calculational lead and marked the transition from human computers to IBM machines (done in April/May ‘44).
  – They used two methods to compute shocks, but only one of them worked well (the shock fitting by Peierls). The other finite difference method produced severe post-shock “wiggles” explained as thermal excitation.

• The first calculations were 1-D and Lagrangian, shocks were tracked (no viscosity, finite differences failed completely till 1948).

• Von Neumann developed a “simple” finite difference method at Aberdeen and published a report on March 20, 1944.
The artificial viscosity paper by Von Neumann and Richtmyer, J. Appl. Phys. 1950

A Method for the Numerical Calculation of Hydrodynamic Shocks

J. Von Neumann and R. D. Richtmyer
Institute for Advanced Study, Princeton, New Jersey
(Received September 26, 1949)

The equations of hydrodynamics are modified by the inclusion of additional terms which greatly simplify the procedures needed for stepwise numerical solution of the equations in problems involving shocks. The quantitative influence of these terms can be made as small as one wishes by choice of a sufficiently fine mesh for the numerical integrations. A set of difference equations suitable for the numerical work is given, and the condition that must be satisfied to insure their stability is derived.

I. INTRODUCTION

In the investigation of phenomena arising in the flow of a compressible fluid, it is frequently desirable to solve the equations of fluid motion by stepwise numerical procedures, but the work is usually severely complicated by the presence of shocks. The shocks manifest themselves mathematically as surfaces on which density, fluid velocity, temperature, entropy and the like have discontinuities; and clearly the partial differential equations governing the motion require boundary conditions connecting the values of these quantities on the two sides of each such surface. The necessary boundary (but preferably somewhat larger than) the spacing of the points of the network. Then the differential equations (more accurately, the corresponding difference equations) may be used for the entire calculation, just as though there were no shocks at all. In the numerical results obtained, the shocks are immediately evident as near-discontinuities that move through the fluid with very nearly the correct speed and across which pressure, temperature, etc. have very nearly the correct jumps.

It will be seen that for the assumed form of dissipation (and, indeed, for many others as well), the Rankine-Hugoniot equations are satisfied, provided the thick-
LA-671, The first description of artificial viscosity written by Richtmyer (only!)

Classified till 8/26/93. In the period right after WWII almost all Lab reports were classified.

The projects Richtmyer was working on in 1947 and 1948 were key to the development of the method. The application was too complex for shock fitting.
Richtmyer published a second report five months later in 1948 (March to August) reporting on numerical experiments.

\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial m}(p + q) = 0 \rightarrow \frac{\partial u}{\partial t} + \frac{\partial}{\partial m}\left(p + \mu \frac{\partial u}{\partial x}\right) = 0
\]

\[
T \Delta S = -\frac{1}{6} G \frac{1}{c^2} \left(\frac{\Delta V}{V}\right)^3 \rightarrow
\]

\[
T \Delta S = \mu \left(\frac{\partial u}{\partial x}\right)^2 \rightarrow \mu \propto (\Delta x)^2 \left|\frac{\partial u}{\partial x}\right|
\]

He uses both the term “fictitious” and “mock” to describe the term, but not “artificial”. All of these are unfortunate in their connotation.
Lax’s contributions have received a great honor - the 2005 Abel Prize*

- Some of the work he was honored for started at Los Alamos and continued while at NYU’s Courant Institute.
  - The work on conservation laws begins in the wake of knowing shock capturing is a workable concept via Von Neumann-Richtmyer’s viscosity.
  - Lax’s efforts form much of the theoretical foundation for CFD today.
  - Basic theory for the analytical and numerical solution of hyperbolic conservation laws.
The beginning of weather/climate is connected to all of this too, through Von Neumann.


**Staggered Grid**

\[
\begin{align*}
  k & : \phi, \psi, u, v \\
  k + \frac{1}{2} & : \rho, \omega \\
  (a) & : \phi, \psi, u, v \\
  (b) & : \phi, \psi, \omega \\
  (c) & : \rho, u, v \\
  p & : \rho, u, v
\end{align*}
\]

ENIAC

First calculation
16x16x(3) mesh
\( \Delta x = 300 \text{ km} \)
48 time steps
\( \Delta t = 30 \text{ minutes} \)
The connection between weather modeling, artificial viscosity and Large Eddy Simulation

• In 1956 a simulation by Norm Phillips of weather over the eastern half of the US for a month was completed and the subject of a meeting at IAS.
  – Late in the simulation the solution began to experience and instability (ringing)
  – It was suggested by Charney that “Von Neumann’s viscosity” might control this ringing.

• Smagorinsky completed the follow-on simulation including this technique
  – This technique became the first Large Eddy Simulation (LES) subgrid turbulence model
  – Smagorinsky’s paper is viewed as the first global circulation model, the precursor to climate models
The concept that works is essential, it provides the faith needed for progress.

- The history of the key ideas involved with shock capturing methods is not well known, but can be revealed now.
  - Quite often, Von Neumann is given sole credit for conceiving of shock capturing and artificial viscosity, Richtmyer played a key role in making shock capturing work practically.
  - The first report on artificial viscosity by Richtmyer was classified for 45 years and it probably shouldn’t have been for that long.
  - The ideas made shock capturing methods useful and provided the impetus for Lax’s incredibly important work.

- These initial ideas laid the foundation upon which the contributions that this symposium honors are built.

- There are other connections that are not well known.
  - Large Eddy Simulation’s canonical model, the Smagorinsky model was originally artificial viscosity
What history tells us

• Von Neumann conceived of shock capturing,
• Richtmyer made it work,
• Lax showed the way forward...
• and we know the rest of the story.