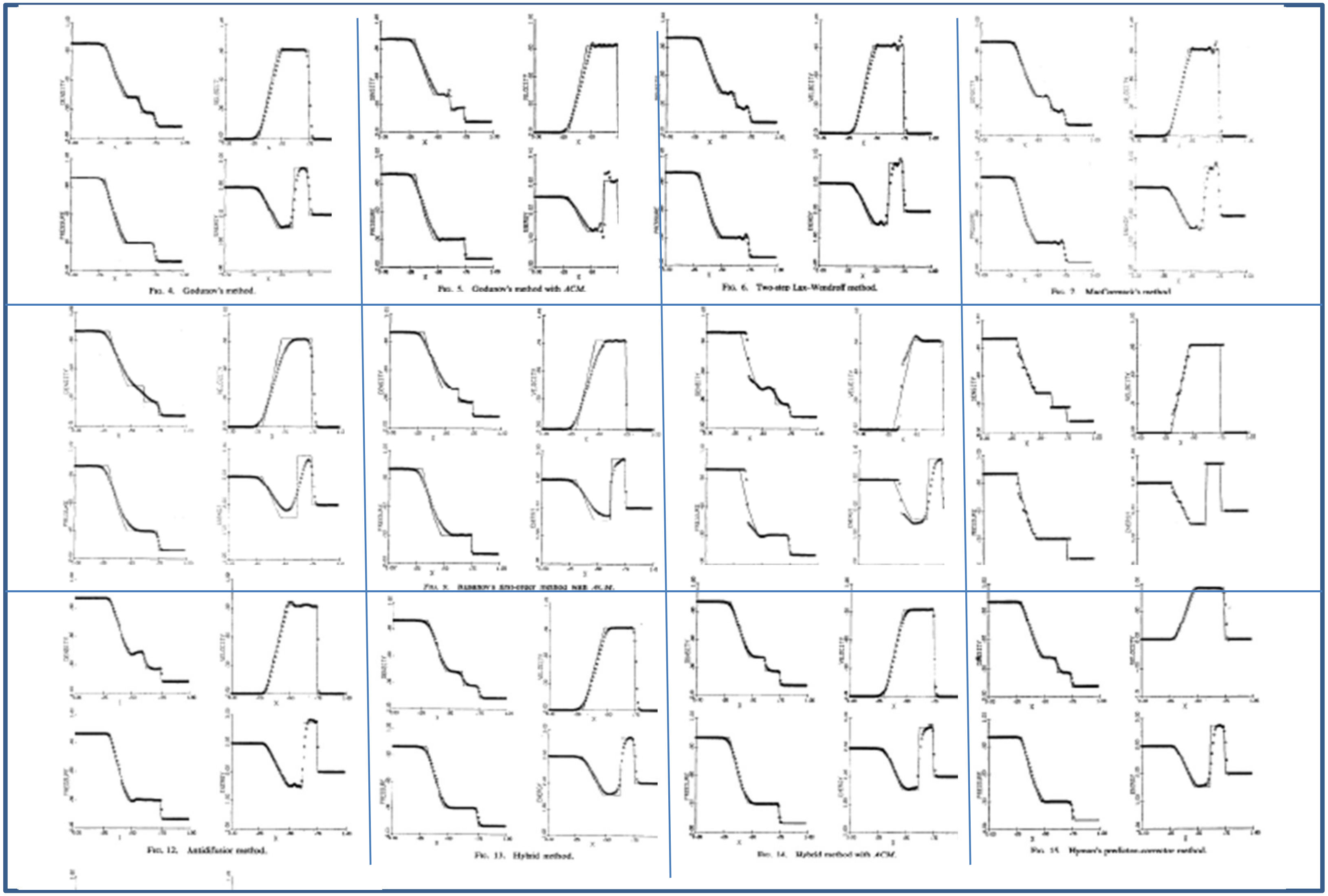
# My First Forty Years

(1974-2013)

Phil Roe

# In 1978, nothing worked!



# The most advanced Euler code available to designers in 1974

Was a second-order finite-volume code based on MacCormack's method

Whereas now.................

It is a second-order finite-volume code not based on MacCormack's method!

# Practical Advances since 1974

- Unstructured grids
- Adaptive grids
- Fast Solvers
- Flux Functions and Riemann Solvers

## In 2013, almost everything works …

- The greatest single advance has been the adoption of nonlinear limiting in some form
- But nothing works particularly well. There is no universally preferred method.

# Why not ???

Is CFD algorithm design a problem having many solutions, all of them having roughly equal merit?

OR

Is there some yet-to-be discovered magic method that will make all others obsolete? One that is high-order, robust and inexpensive?

# What is a high-order scheme?

- To a mathematician, a scheme for which the truncation error is proportional to $h^p$ as the mesh size tends to zero
- To an engineer, A scheme for which the error remains acceptable as the mesh gets coarser

# The order of difficulty p, of a scientific problem

Is the smallest number of good ideas required to resolve it. (It is the height at which the fruit hangs!)

High-order methods are high-order in more than one sense.

# Why is CFD difficult?

- Actually, why is fluid dynamics difficult ?
- At a small scale, fluid dynamics is rather easy
- At a large scale, fluids are very lightly constrained, and behavior can become arbitrarily complex
- The difficulty is to bridge the scales

# Doing it nature's way

- Model the PDES as simply as possible (e.g. Lax-Friedrichs)
- Wait for the complexity to emerge on a fine enough grid
- All the difficulty comes from trying to see the complexity on coarse grids
- That is, of necessity, a nonlinear business.

# What Did We Learn from the ICASE equation?

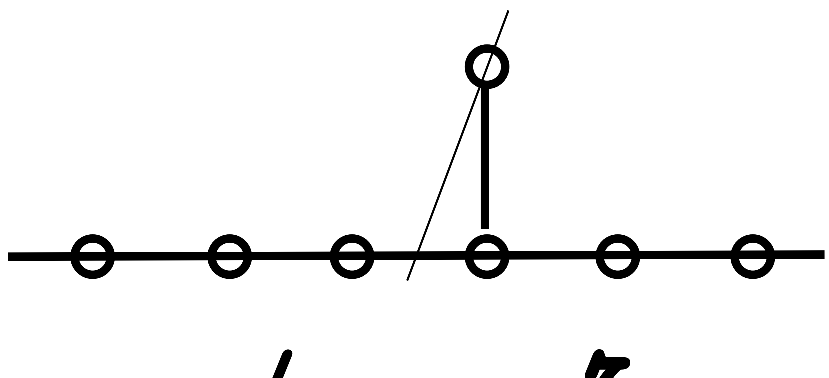
$$\mathbf{u_t + a u_x = 0}$$

- High order implies nonlinearity (Godunov 1957)
- Stability requires upwinding (Iserles, 1982)
- Odd order is better than even Order (Hedstrom 1974, Bouche et.al. 2003)

# Iserles' theorem

(IMAJNA, 1984)

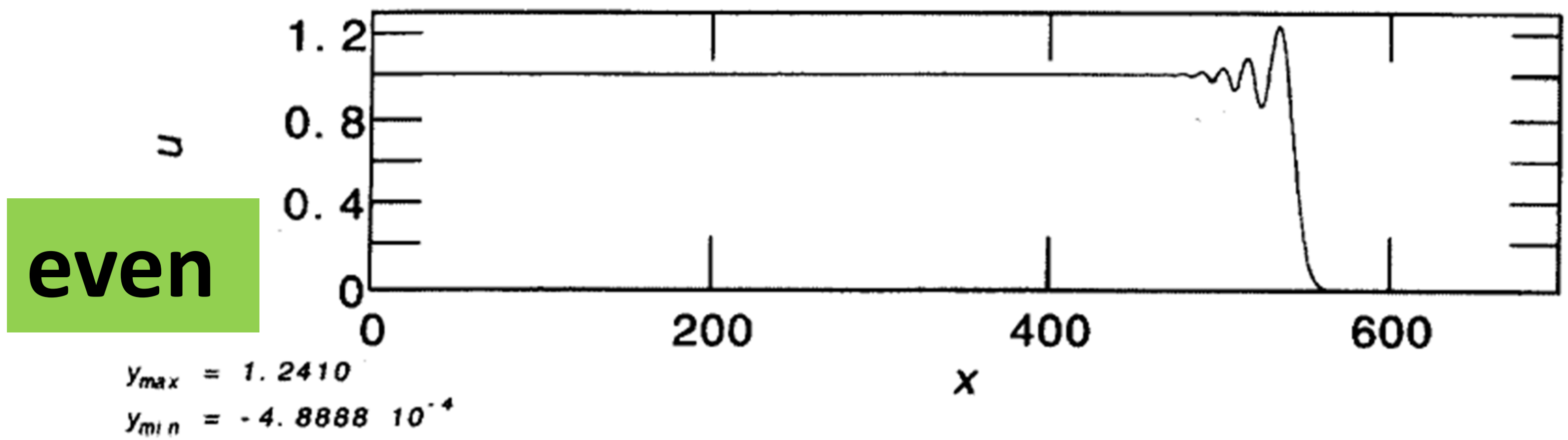
Consider a stencil having $\ell$ intervals to the left and $r$ intervals to the right.

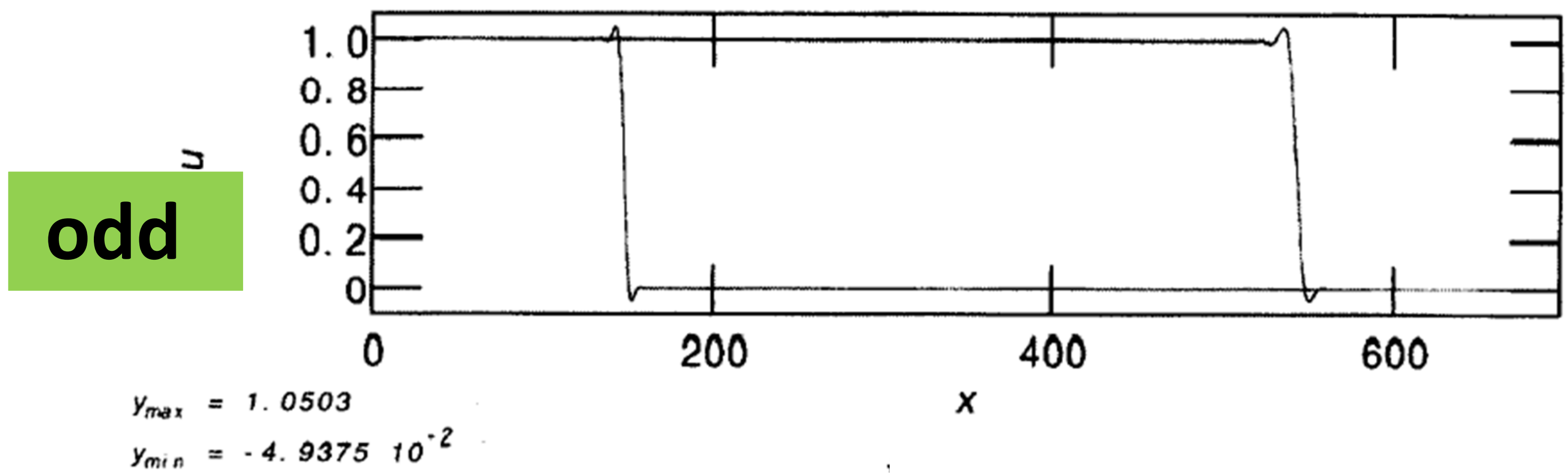



Polynomial interpolation with maximal accuracy is only possible if $\ell - r =$ $0, 1, 2 \ (a > 0)$

# Odd and even profiles of a discrete step

Hedstrom (Math Comp, 1975), Bouche et al (App Math Lett, 2003)



# On Simplicity

A theory should always be as simple as possible, but no simpler

(Einstein)

Seek simplicity, but never trust it

(Eddington}

The ICASE equation says nothing about oblique waves, or about vorticity, or about preserving symmetry

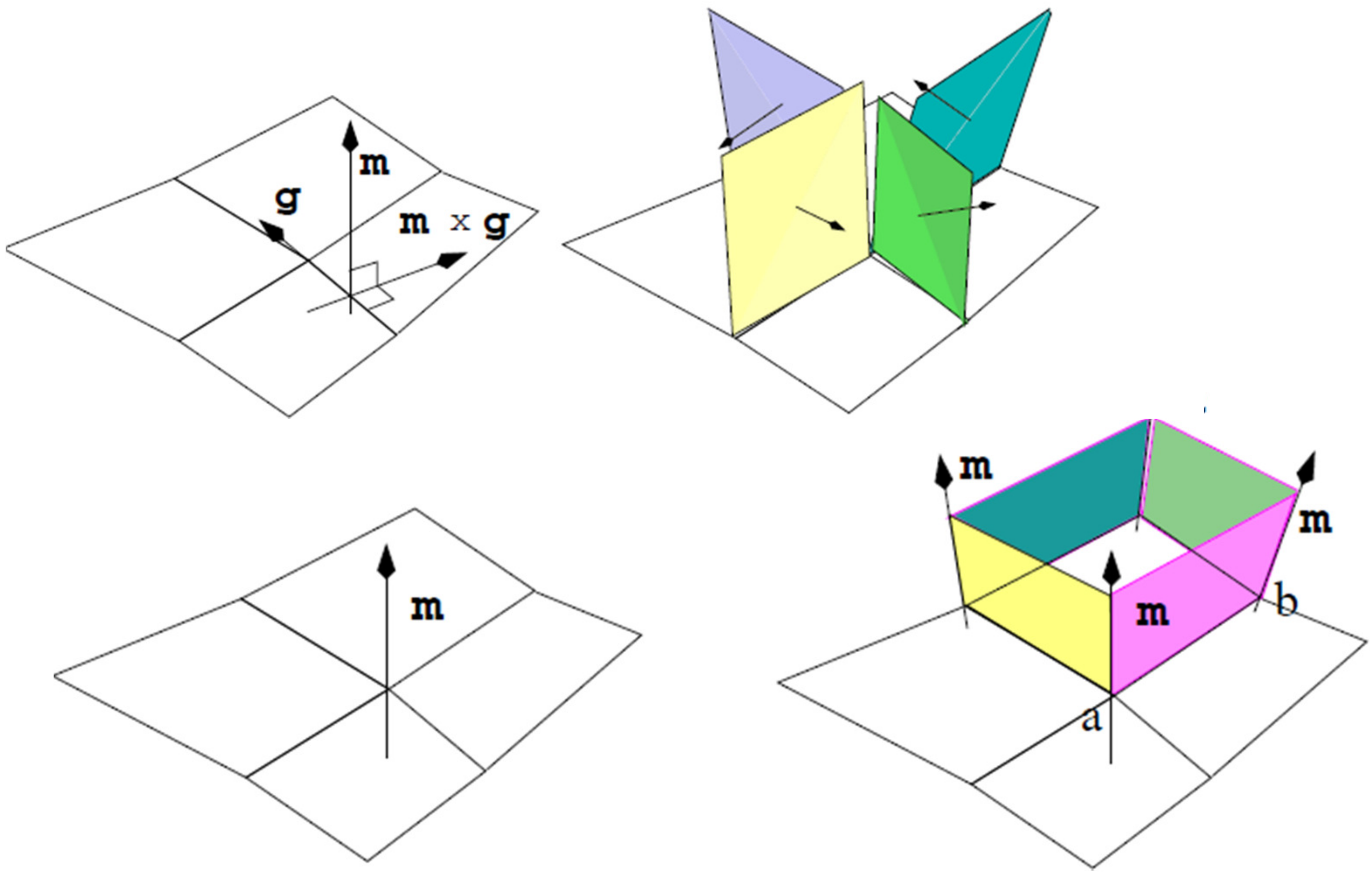
# The new ICASE equation ?

Yields information on all of these points

$$\begin{aligned} \partial_t p + \mathrm{div}\, \mathbf{v} &= 0 \\ \partial_t \mathbf{v} + \mathrm{grad}\, \mathrm{p} &= 0 \end{aligned}$$

For example, vorticity is preserved only if fluxes are evaluated at vertices (Morton and Roe, sisc, 2001; Mishra and Tadmor, 2011) prompting the question, what is a flux?

# An example from front propagation



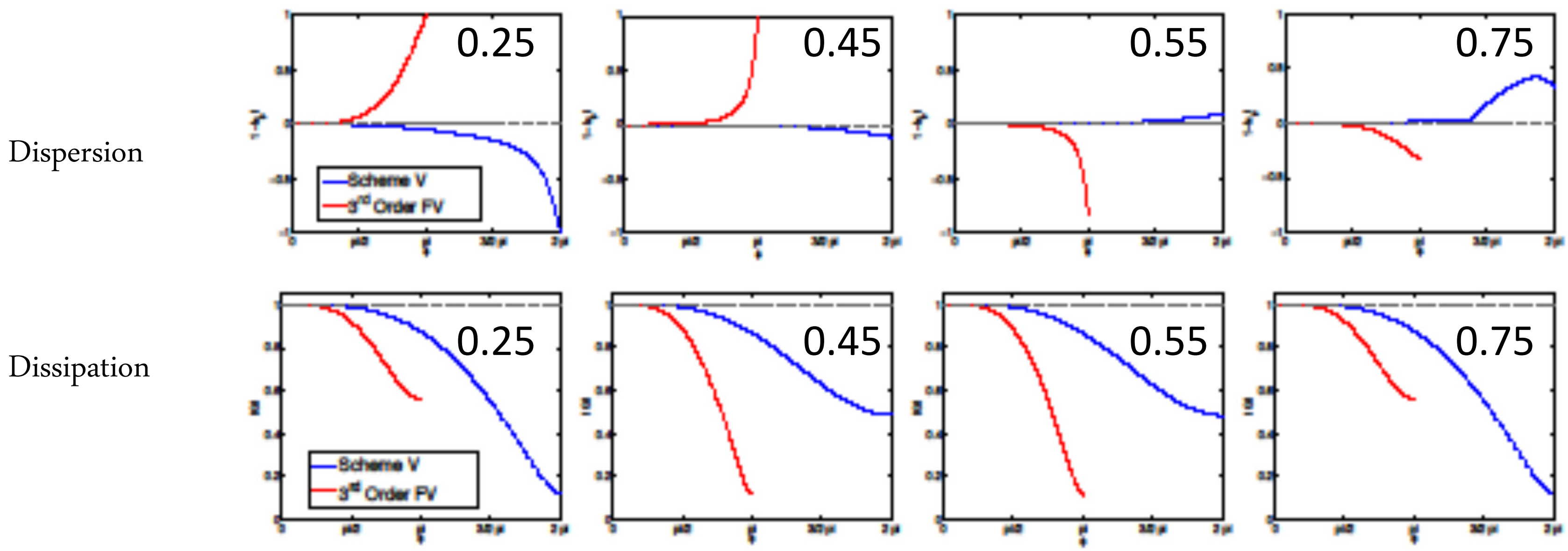

Varadarajan and Roe, AIAA Fluid dynamics 2011

# What are the right questions ?

- New extrema have to be accepted, but when ?
- What would define an “optimum” low-order scheme?
- A Lax-Wendroff scheme has four free parameters; what are they good for?
- How do you avoid “mesh-imprinting”?

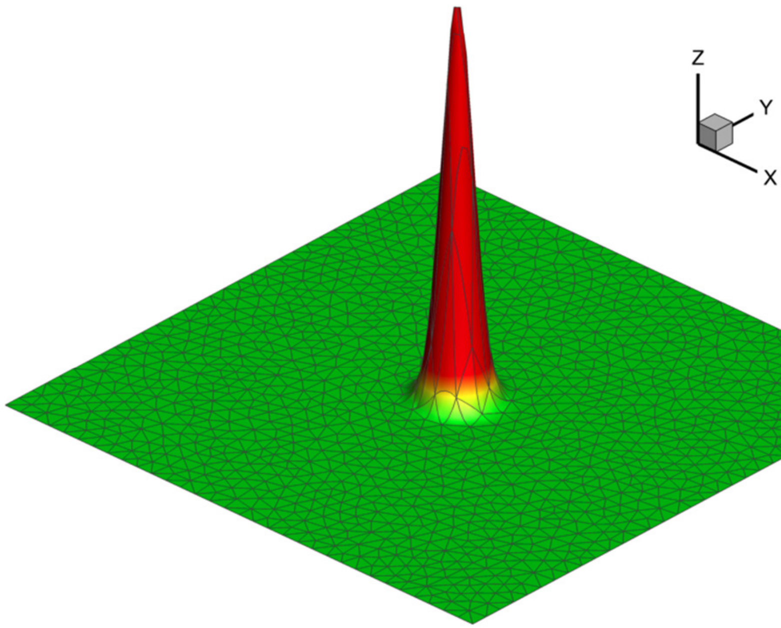
# Revisiting Scheme V

- The fluxes of a finite-volume scheme can be regarded as independent unknowns.
- This doubles the resolvable frequencies, and raises the accuracy to third-order.



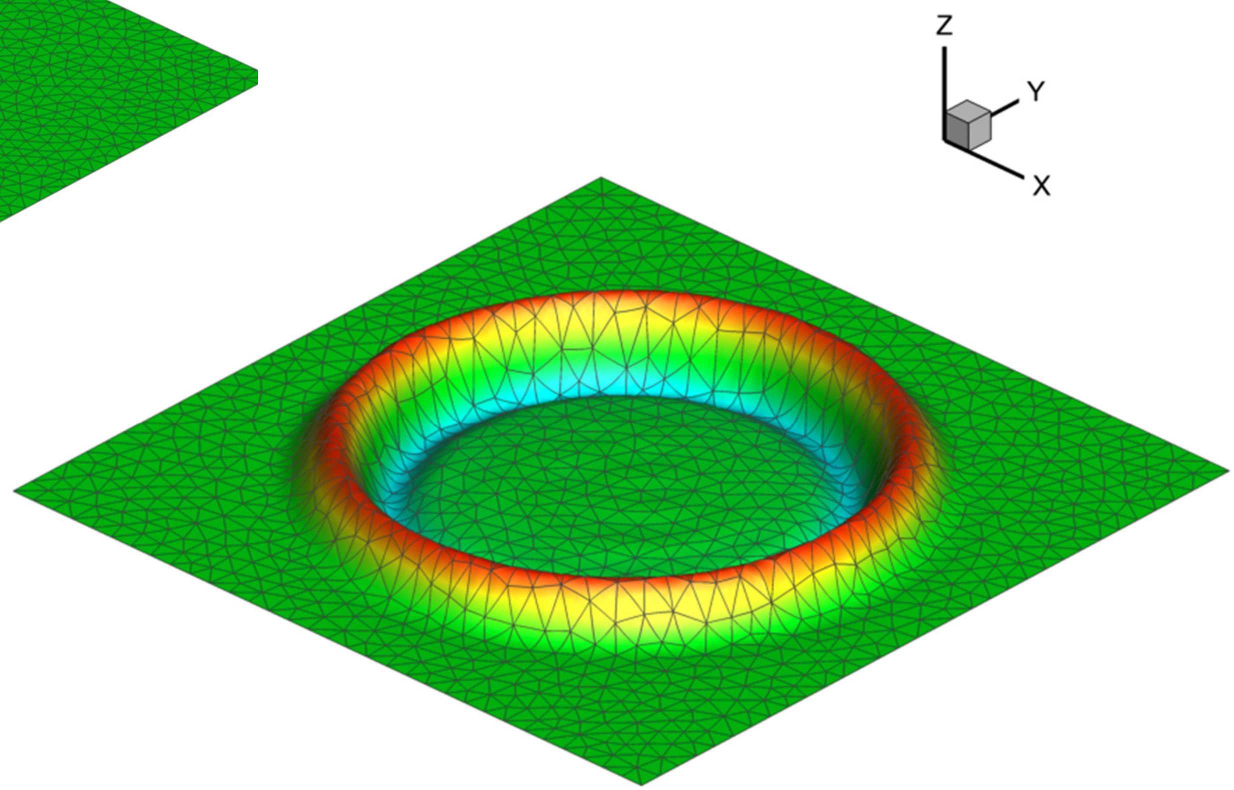

Comparison with a classical finite-volume scheme

# Linearized Euler on an Unstructured Grid




Piecewise quadratic elements updated by exact solution to locally linearearized problems.

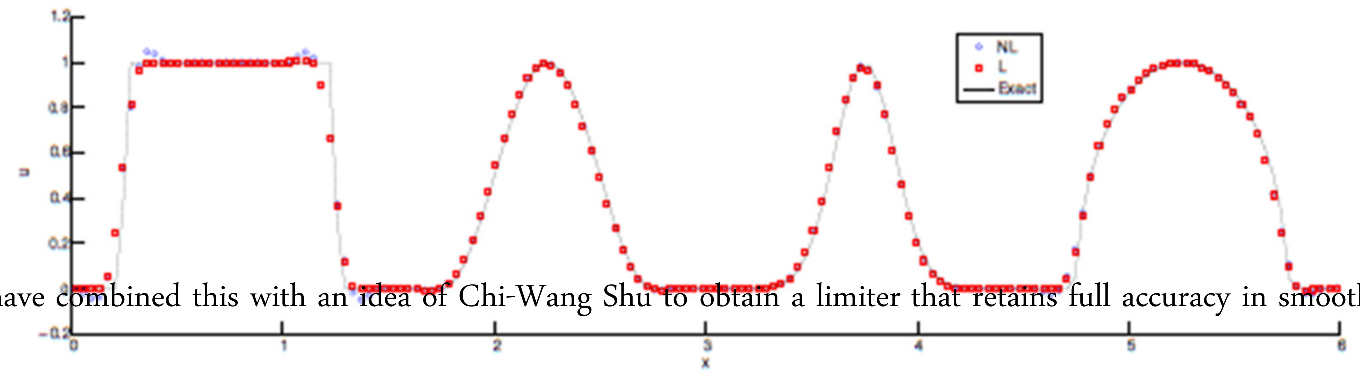
Calculations of a spreading pressure pulse on a coarse unstructured grid



# Limiting in Time

We need to measure our confidence in a reconstruction by comparing it with alternatives.

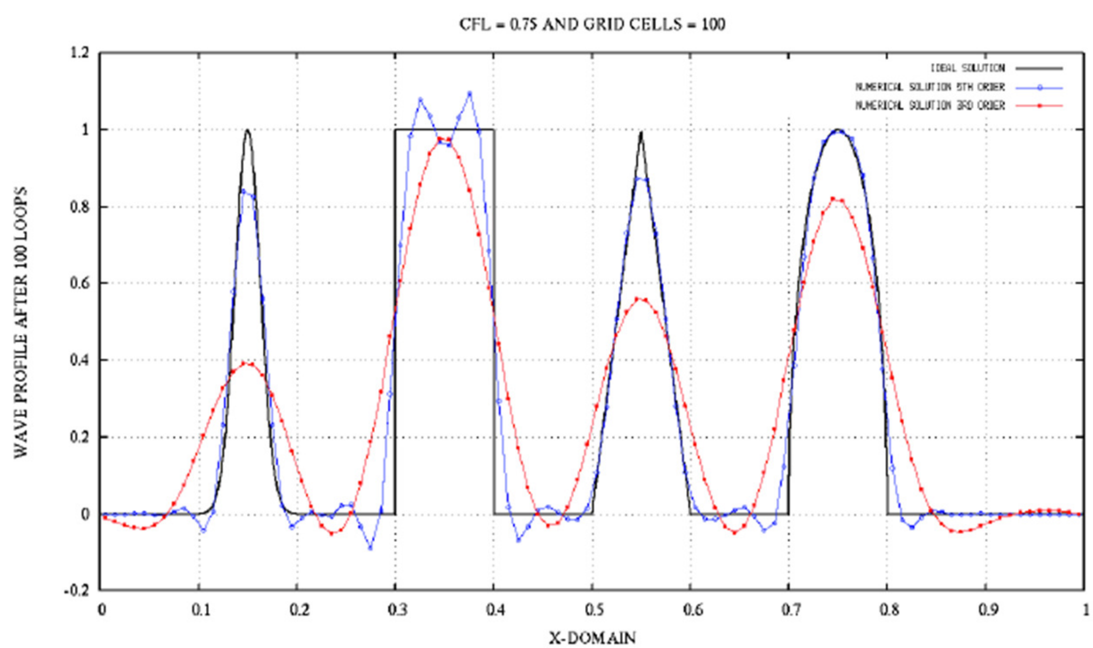
- Usually, we do this by comparing with neighboring reconstructions (Slope-limiting, Weno)
- We can also compare with previous reconstructions in the same cell.




- We have combined this with an idea of Chi-Wang Shu to obtain a limiter that retains full accuracy in smooth regions

# Higher and higher?

- Another fifth-order version of Scheme V comes from storing also the gradients at interfaces (3 dof/cell)




Results after each wave has propagated one thousand times its own length, defined by ten mesh intervals.

Today

Is the first day

of the next forty years