A Better Understanding of the Earth System Through Advances in CFD

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Part 1

Atmospheric Models and the Need for Resolution
Anatomy of an Atmospheric Model

Dynamical Core

Responsible for solving the fluid equations on large-scales (~100km or greater horizontal resolution)
CESM Atmosphere Overview

- **CSM-1 (1996):** CCM3 Eulerian (spectral transform) dynamical core (T42) 2.8 degree horizontal resolution (26 model levels)

- **CCSM-2 (2002):** CAM2 Eulerian dynamical core, (T42) 2.8 degree horizontal resolution (26 model levels)

- **CCSM-3 (2004):** Eulerian dynamical core, 1.4 to 2.8 degree horizontal resolution (26 model levels) in CAM3


- **CESM-1.1 (2012):** Spectral element atmospheric dynamical core (default), 1 degree horizontal resolution (30 model levels) in CAM5.2
The cubed-sphere grid is obtained by placing a cube inside a sphere and “inflating” it to occupy the total volume of the sphere.

- No polar singularities
- Grid faces individually regular
- Some difficulty at panel edges
- Non-orthogonal coordinate lines
Looking Back and Moving Forward

The computing power of modern supercomputing systems are growing at an exponential pace.

This motivates the need for atmospheric models that can harness these large-scale parallel systems.

Higher resolution plus numerical method locality is a simple approach for maximizing utilization.

Note logarithmic scale → Exponential growth!
Looking Back and Moving Forward

CAM4 0.25° (28km) scalability
IBM GB/P “Intrepid”

Spectral element dynamical core (using a local finite-element method) achieves near-perfect scalability to 1 element per core (86000 cores) with peak performance 12.2 SYPD

Full CESM runs ~ 50% slower due to other components

Source: Mark Taylor, correspondence
Looking Back and Moving Forward

Computational power doubles approximately every 1.2 years.

To obtain a factor of 2 horizontal refinement, numerical models require 8x the computational power.

Doubling of horizontal resolution every 3.6 years?

In reality slower due to increased vertical resolution, more tracers and more complicated parameterizations.
Climate Model Resolution

1990

FAR
~500 km (T21)

1995

SAR
~250 km (T42)

2001

TAR
~180 km (T63)

2007

AR4
~110 km (T106)

AR5 ~ 25km? (2014)
Atmospheric Features by Resolution

Characteristic Time Scale
- 10 days
- 1 day
- 1 hr
- 10 min
- 1 min
- 1 s

Characteristic Length Scale
- 10 cm
- 10 m
- 1 km
- $10^2$ km
- $10^4$ km

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(25km) Resolution of tropical cyclones; experimental global climate resolution
Atmospheric Features by Resolution

(10km) Transition to non-hydrostatic scales; Next-generation NWP resolution
Why Higher Resolution?

- Improved resolution of land-surface processes (snowpack, runoff)
- Resolution of transient eddies (synoptic-scale frontal systems, local convective systems)
- Resolution of extreme weather events
- Improvement in representation of geographic features (mountain ranges and islands)

The California coastal ranges have a dramatic effect on regional climate which is poorly captured in current climate models.
MCore: Global Model

Started from a collaboration between atmospheric science and aerospace at the University of Michigan:

High-order finite-volume methods for the shallow-water equations on the sphere

Paul A. Ullrich*,a, Christiane Jablonowskia, Bram van Leerb

**Flow Over Topography**

Flow over topography – Fluid Height (m) – Day 0 Step 0

Baroclinic Instability on the Sphere

MCore

GFDL FV3

CAM-EUL

CAM-SE
**MCore/Chombo: Global Model**

MCore/Chombo is a next-generation non-hydrostatic atmospheric dynamics model for computing solutions of the flow equations on the sphere:

- Collaboration between UC Davis and Lawrence Berkeley National Lab (Phillip Colella, Hans Johansen, Bill Collins)

- The finite-volume approach is used to maintain conservation and avoid numerical artifacts due to spectral ringing.

- Designed to scale well on parallel systems, using a cubed-sphere grid to enforce grid uniformity.

- Utilizes horizontal-vertical splitting to avoid issues caused by the relatively fine vertical spacing.
Mesh Refinement

Since the MCore / Chombo model uses non-conformal grids, the grid may be actively adapted to follow features of interest.
Ongoing Work on Multi-Res Models

Variable resolution modeling using the Community Earth System Model (CESM) is ongoing. Efforts are currently directed at determining changes in tropical cyclone counts associated with the changing climate.

Figures courtesy Colin Zarzycki (2012)
Hurricane Sandy

Variable resolution CAM-SE showing wind speed of an evolving hurricane, using data from Friday, October 26.

We are on the cutting edge of having the capability of running live hurricane forecasts in the global domain.

Improved performance is expected with an adaptive meshing approach.

Movie courtesy Colin Zarzycki (2012)
Dessert

Numerical Methods and the Treatment of Atmospheric Waves

Berkeley Lab

UC Davis

University of California
Looking at Numerical Methods…

- How do we approach this problem?
- What are the features of the atmosphere that make it a unique modeling domain?
- Pitfalls of a poor choice: Wasted computing time, inaccurate solutions, poor convergence, lack of flexibility
Why Waves?

• To a close approximation, the atmosphere is in a state of geostrophic and hydrostatic balance.

• For atmospheric flows, departures from geostrophic balance are approximately linear. The Mach number of these flows is generally much less than one, and shock waves are not present.

• In this regime, the dynamical character of the fluid is dominated by wave motion.

• As a result, the correct treatment of linear waves is intrinsic to any accurate approach for modeling the equations of motion.
Important Questions

• What are the *shortest waves* which can be considered “resolved” for a particular numerical method?

• What is the effect of *increasing the order of accuracy* of a numerical discretization on its treatment of waves?

• For a given order of accuracy, which numerical methods offer the *best treatment of wave-like motion*?

• For a given error level, which numerical method and order of accuracy is the *most computationally efficient*?
Numerical Methods of Interest

- (UFV) Upwind Finite-Volume
- (CFV) Central Finite-Volume
- (stFV) Staggered Finite-Volume
- (SFV) Spectral Finite-Volume
- (DG) Discontinuous Galerkin
- (DG*) Mass-Lumped Discontinuous Galerkin
- (SEM) Spectral Element Method
- (MB-SEM) Modified Basis Spectral Element Method
- (FR) Flux Reconstruction Methods

Missing: Mixed Finite-Element Methods (UK Met Gung-Ho?)
1D Wave Equation

Two coupled equations:

\[
\frac{\partial u}{\partial t} + \frac{\partial \Phi}{\partial x} = 0 \\
\frac{\partial \Phi}{\partial t} + \frac{\partial}{\partial x}(\Phi_0 u) = 0
\]

Used to model small amplitude gravity waves in a shallow ocean basin.
The evolution equation for the degrees of freedom within each element can then be written in the general form:

\[
\frac{\partial u_j}{\partial t} + \frac{1}{\Delta x_e} \sum_{\ell=-N_g}^{N_g} A^{(\ell)} \Phi_{j+\ell} = \frac{\sqrt{\Phi_0}}{\Delta x_e} \sum_{\ell=-N_g}^{N_g} C^{(\ell)} u_{j+\ell}
\]

\[
\frac{\partial \Phi_j}{\partial t} + \frac{\Phi_0}{\Delta x_e} \sum_{\ell=-N_g}^{N_g} B^{(\ell)} u_{j+\ell} = \frac{\sqrt{\Phi_0}}{\Delta x_e} \sum_{\ell=-N_g}^{N_g} D^{(\ell)} \Phi_{j+\ell}
\]

**Discrete Form**

**Advective Terms**

**Diffusive Terms**

\(N_g\) is the stencil width (coupling to neighbors)
Shortest Resolved Wave

Idea: The “resolution” of a numerical method is the shortest wavenumber that can be considered resolved to a specified error level.

Claim: Allows for an apples-to-apples comparison of numerical methods.

Note: Compare dispersive (phase speed) and diffusive (damping) errors separately.
**Shortest Resolved Wave**

Shortest Resolved Wave (Dispersive Limit)
(with implicit diffusion)

![Graph showing the order of accuracy with implicit diffusion](image)

Order of Accuracy

As expected: Shorter waves are resolved as resolution increases

However, diminishing returns beyond 5th order!
Shortest Resolved Wave

Order of Accuracy

Best phase speed for DG* and SFV occurs for the 5th order accurate schemes.

“Spectral gap” clearly hurts the performance of SEM
Spectral Element Method (SEM)

Gaps appear in the SEM spectrum at 4th order and above which worsen with order

SEM does not have an implicit diffusion term. Application of these methods in practice requires augmenting the scheme with explicit diffusion.
Modified Basis SEM

Modified Basis SEM (MB–SEM) eigenstructure

Gaps are removed by adjusting the SEM interior node locations.

Modified basis method adjusts nodes to minimize the presence of gaps in the spectrum.
**Shortest Resolved Wave (Diffusion)**

- **Order of Accuracy**
- Shortest Resolved Wave (Diffusive Limit)
  - (with implicit diffusion)

Diffusivity of methods for longer wavenumbers is easily ranked.

At this error level, waves can be considered resolved at approximately 4 $\Delta x$ for highest order methods.

- **Diffusive methods seem to have better behaved MRW limit.**

**Operational models:**
- MPAS (2$^{\text{nd}}$ order stFV) 10 $\Delta x$
- CAM-FV (4$^{\text{th}}$ order CFV) 8 $\Delta x$
- CAM-SE (4$^{\text{th}}$ order SEM) 6-9 $\Delta x$
**Maximum Courant Number**

Maximum Courant Number (CFL Condition) with SSPRK53

(with implicit diffusion)

(with no implicit diffusion)

Monotone decay of maximum Courant number with order of accuracy.
FLOPs per DOF

FLOPs / Memory Reads per Node

(with implicit diffusion)

FLOPs / Memory Reads per Node

(with no implicit diffusion)

Includes cost from diffusive operator. SFV and DG have identical FLOP cost.

All four schemes have the same linear increase in cost with order.
Approximate Equal Error Cost

Combining all of the information we have accumulated about these schemes:

\[
\langle \text{Approximate equal error cost} \rangle = \langle \text{Average non-zeros per DOF} \rangle \times \langle \text{Shortest resolved wavelength} \rangle \\
\times \langle \text{Number of RK stages per time step} \rangle \\
/ \langle \text{Maximum stable Courant number} \rangle
\]

This error measure assumes we can change the grid resolution so as to obtain a fixed error norm.
Approximate Equal Error Cost

(a) RK4 (with implicit diffusion)

Minimum cost for SFV, DG and DG* at 3\textsuperscript{rd} order. Minimum cost for UFV at 5\textsuperscript{th} order.

(b) SSPRK53 (with implicit diffusion)

Costs are lower for these schemes when a SSPRK53 time integrator is used.
Approximate Equal Error Cost

(c) RK4 (no implicit diffusion)

(d) SSPRK53 (no implicit diffusion)

Costs are lower for these schemes when a RK4 time integrator is used.

Cost is largely a monotone increasing function of order.
Some Tidbits

• Results show an interesting intercomparison of these methods to the case of smooth problems.

• Compact numerical methods from the finite-element family exhibit competitive treatment of wave-like motion to finite-volume methods.

• Diminishing returns above 5th order for many methods; even for high-order methods, waves are not resolved below 4 Δx.
Some Tidbits

Equivalent dispersion properties were found for a number of methods:

• Mass-lumped DG* and Huynh’s Flux-Reconstruction with $g_2$
• Spectral Finite-Volume and Huynh’s Flux-Reconstruction with $g_{Ga}$
• Spectral Element Method, DG on GLL nodes and Spectral Finite-Volume combined with a central flux
Epilogue

The Future of Global Atmospheric Models
Summary

• Model resolution continues its march forward, leading to resolution of previously unresolved physical processes.

• Under dispersion analysis, it has been shown that compact finite-element methods have competitive wave capturing properties to single-moment methods. Regardless of the choice of numerical method, third- or fourth-order accuracy is desirable.

• Work is ongoing globally to develop variable resolution models (using both static and adaptive mesh refinement) to better answer questions about changing regional climate.

• New software infrastructure is needed to support these changes.
Finding 3.1: “Climate models are continually moving toward higher resolutions via a number of different methods in order to provide improved simulations and more detailed spatial information; as these higher resolutions are implemented, parameterizations will need to be updated.”

“New model parameterizations need to be designed to work well across the range of model grid spacings and time steps over which they may be used, which is often not straightforward.”
Thank You