Looking Forward to Looking Back

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Looking Back is Good

- It anchors your plans for the Future
- It makes you discover missed opportunities

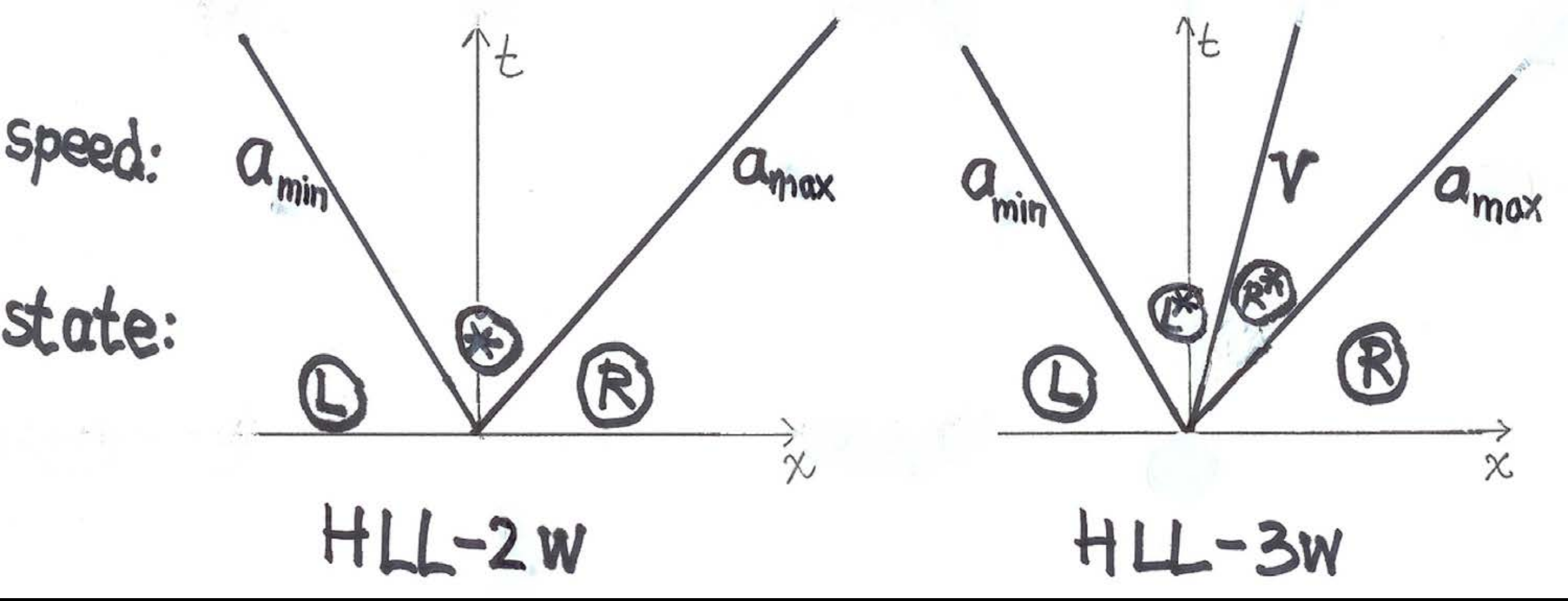
HISTORY OF CFD: PART II

Top level: Jay Boris, Vladimir Kolgan, Bram van Leer, Antony Jameson
Ground level: Richard Courant, Kurt Friedrichs, Hans Lewy, Robert MacCormack, Philip Roe, John von Neumann, Stanley Osher, Amiram Harten, Peter Lax, Sergei Godunov

1980
Year of the Riemann Solver

- Osher (all waves)
- Harten-Lax (2/3 waves)
- Van Leer (Euler flux splitting)
- Roe (all waves)

Harten-Lax-Van Leer Riemann Solvers



The Expression for V (I)

- **Start with**

  HSCL: $u_t + f_x = 0$**, with**

  $$f_u = \mathbf{A}$$

- **Transform to entropy variable** w**, with**

  $w_u = \mathbf{P}$**: symmetric positive-definite,**

  $f_w = \mathbf{B}$**: symmetric.**

- **Then:** $f_u = f_w \cdot w_u = \mathbf{BP} \Rightarrow \mathbf{A} = \mathbf{BP}$

The Expression for V (II)

- Define Roe-type averages of $\mathbf{A}, \mathbf{P}, \mathbf{B}$ in the space of entropy variables.
- For instance:

$$\Delta f = \int_0^1 \frac{d}{d\theta} f(w_L + \theta \Delta w) d\theta$$

$$= \int_0^1 f_w \Delta w \, d\theta = \int_0^1 \mathbf{B} \Delta w \, d\theta = \bar{\mathbf{B}} \Delta w.$$

- Similarly:

$$\Delta w = \bar{\mathbf{P}} \Delta u, \Delta f = \bar{\mathbf{A}} \Delta u \Rightarrow \bar{\mathbf{A}} = \bar{\mathbf{B}} \bar{\mathbf{P}}.$$

The Expression for V (III)

Now

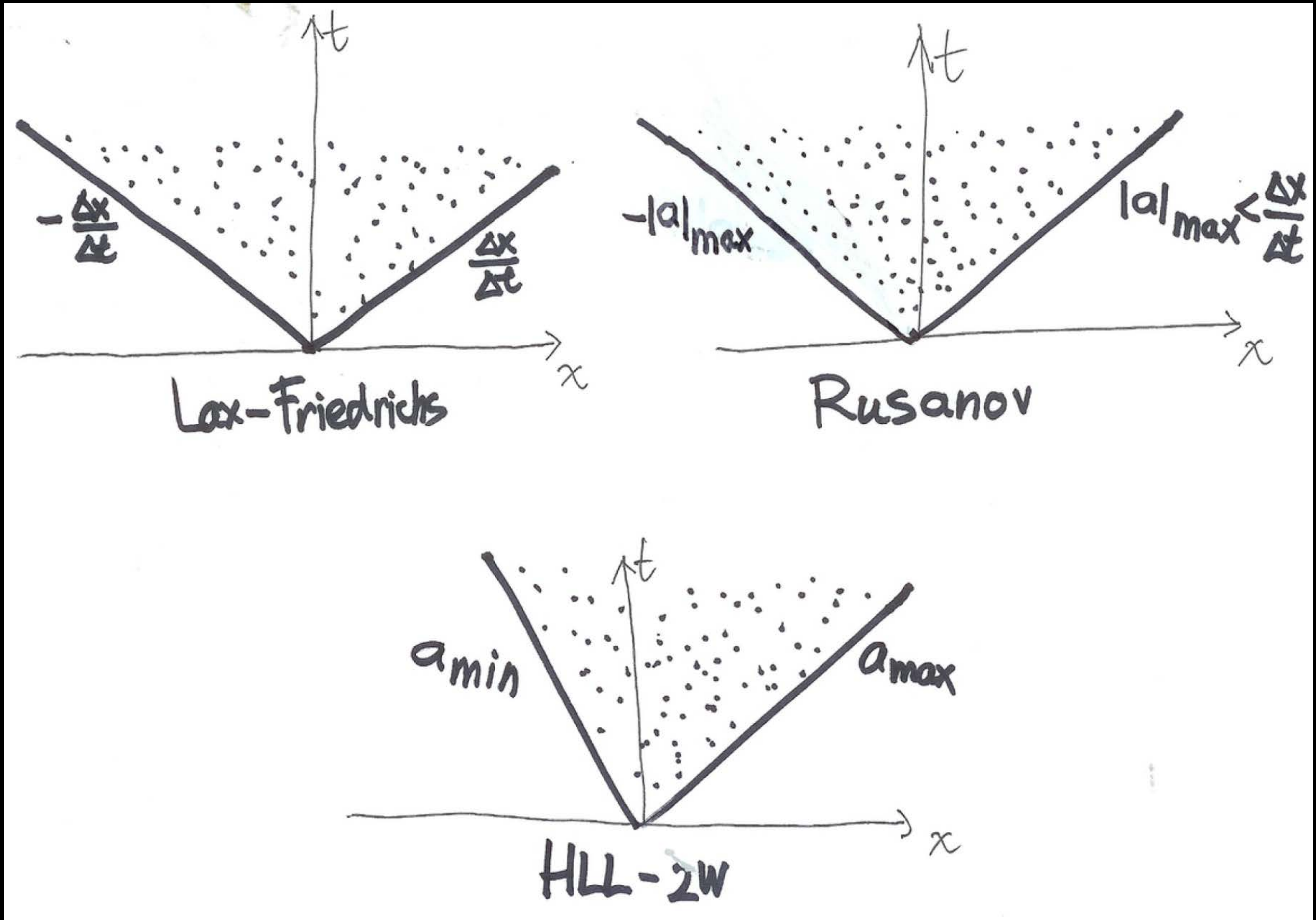
$$V = \frac{\Delta w \cdot \Delta f}{\Delta w \cdot \Delta u}$$

$$= \frac{\bar{\mathbf{P}}\Delta u \cdot \bar{\mathbf{B}}\bar{\mathbf{P}}\Delta u}{\bar{\mathbf{P}}\Delta u \cdot \Delta u} = \frac{(\bar{\mathbf{P}}^{1/2}\Delta u) \cdot \bar{\mathbf{P}}^{1/2}\bar{\mathbf{B}}\bar{\mathbf{P}}^{1/2}\left(\bar{\mathbf{P}}^{1/2}\Delta u\right)}{(\bar{\mathbf{P}}^{1/2}\Delta u) \cdot (\bar{\mathbf{P}}^{1/2}\Delta u)}$$

$\Rightarrow V$ lies in the range of eigenvalues of the symmetric matrix $\bar{\mathbf{P}}^{1/2}\bar{\mathbf{B}}\bar{\mathbf{P}}^{1/2}$.

But: $\bar{\mathbf{P}}^{1/2}\bar{\mathbf{B}}\bar{\mathbf{P}}^{1/2} = \bar{\mathbf{P}}^{1/2}(\bar{\mathbf{B}}\bar{\mathbf{P}})\bar{\mathbf{P}}^{-1/2} = \bar{\mathbf{P}}^{1/2}\bar{\mathbf{A}}\bar{\mathbf{P}}^{-1/2}$ similar to $\bar{\mathbf{A}}$, hence with the same eigenvalues.

HLL-2W is Upwind Biased



Loss of Knowledge (I)

- 1981 Harten & Lax (2w, 3w)
- 1983 Harten, Lax & Van Leer (2w, 3w)
- 1988 Einfeldt (HLLR: 2w, Euler)
- 1991 Einfeldt, Munz, Roe, Sjögreen (HLLE: 2w, Euler)

Loss of Knowledge (II)

- 1994 Toro, Spruce, Speares (HLLC: 3w, Euler)
- 1997, 1999, 2009 Toro's book (HLLC)
- 2002 Linde (HLLL: 3w)
- 2005 Luo, Baum, Löhner (HLLC for all Mach number)

Loss of Knowledge (III)

"… As pointed out by Harten, Lax and van Leer themselves, this defect of the HLL scheme may be corrected by restoring the missing waves. Accordingly, Toro, Spruce and Speares proposed the so called *HLLC scheme*, where *C* stands for *Contact*. In this scheme, the missing middle waves are put back into the structure of the approximate Riemann solver."

A Coat of Arms for All