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Isogeometric analysis
Test case C1.1
Internal inviscid flow over a smooth bump

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1 Code description

The isogeometric analysis is an extension of classical finite element analysis. It has been first introduced by Hughes [1]. B-spline- or Nurbs-based CAD (Computer-Aided Design) defines control points which allow to describe accurately complex geometries. The main idea of isogeometric analysis is to use these control points as degrees of freedom for the flow computation, and B-splines or Nurbs functions as basis functions for the finite element method. The degrees of the Nurbs functions can be chosen so as to obtain a resulting high order method. A pseudo-time stepping scheme (here backward Euler) is used to march the solution up to a steady state. An implicit finite-element approach is finally derived from the classical Galerkine formulation. For every pseudo-time step, a linear system is inverted by using a preconditioned $GMRES(k)$ algorithm.

2 Case summary

The test case which is studied is C1.1 (internal inviscid flow over a smooth bump). L_2 norm of the density residual is used to monitor convergence. Steady state is assumed if the initial residual is dropped by 10 orders of magnitude. Degrees of freedom are here the total number of control points.

3 Meshes

The initial mesh is built from a Nurbs-based 21×5 control points CAD. The initial degree of the Nurbs is 2. The resulting mesh accurately fits with the proposed bump geometry. h-refinement (mesh refinement) corresponds to an increase in the number of control points. The technique described in [1] is used to increase the number of control points (degrees of freedom) while keeping exactly the shape of the bump.

4 Results

Here the error is computed as followed :

$$error_{L2} = \sqrt{\frac{\int_\Omega \left(\frac{p/\rho^\gamma - p_\infty/\rho_\infty^\gamma}{p_\infty/\rho_\infty^\gamma}\right)^2 dV}{\int_\Omega dV}} \tag{1}$$

In Fig. 1, $error_{L2}$ is represented vs. $\sqrt{nDOFs}$ for two different degrees for Nurbs basis functions ($p = 2$ and $p = 3$). Second and third order slopes are shown too.

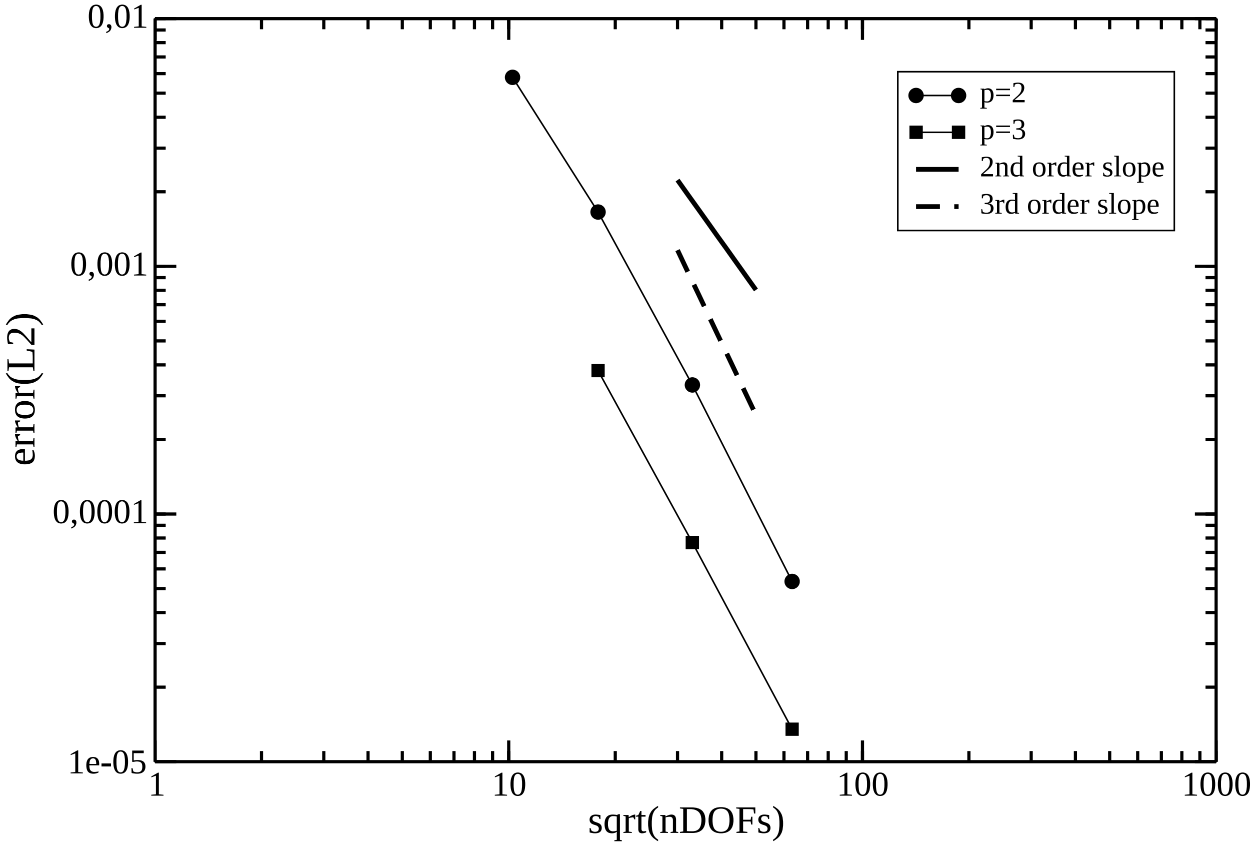

Figure 1: $error_{L2}$ vs. $\sqrt{nDOFs}$. $p = 2$ and $p = 3$ are represented as degrees for the Nurbs functions.

References

[1] TJR Hughes, JA Cottrell, and Y. Bazilevs. Isogeometric analysis: Cad, finite elements, nurbs, exact geometry and mesh refinement. *Computer methods in applied mechanics and engineering*, 194(39-41):4135–4195, 2005.