**1st International Workshop on High-Order CFD Methods**

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# Abstract for the Onera NXO method

# Case 3.5 Taylor-Green Vortex.

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## 1. Code description

_ Discretization by cell-centered Finite Volume Method,

Upwind-biased convective scheme based on characteristic splitting of the conservative variables at the cell interface, no added artificial dissipation operators,

Evaluation of left and right conservative variables as surface averages on the interface interpolated from the volume averaged conservative variables inside cells. The interpolations are based on weighted least-square polynomial reconstructions inside partially biased stencils (collection of cells centered on the left, respectively right cell on either side of the interface).

Centered scheme for the evaluation of gradients for the diffusive fluxes,
Evaluation of centered conservative variables gradients as surface averages on the interface based on a weighted least-square polynomial reconstruction inside a face-centered stencil (union of the 2 stencils used for the convective operator). These reconstructions are done in the pre-processor and provide sets of linear interpolation coefficients for the discrete conservative variables fields and their gradients.

The degree of the reconstructed polynomial is the highest enabled by the number of cells in the stencil (number of monomials *1.5), depending on the successive neighbours insertion (ref 1.).

_ Relevant solvers
Time accurate solutions either

- Non-linear implicit by dual time-stepping (3rd order in real time, Explicit RK for pseudo-time inner iterations), or
- Explicit Runge-Kutta options from 3 to 6 stages (DNS case 3.5).

_ High-order capability
K-exact reconstruction coded and verified up to a 4th degree full base of monomials, for quite regular patterns of unstructured triangles, tetrahedra, hexahedra.
Assertion of the spatial order of convergence for the transport by the upwind-biased convective operator of a sine-wave in a 3-periodic cube, cfl = 1 (fig 1).
4th degree full base polynomial reconstruction => Order of spatial convergence = 4.8
3rd degree full base polynomial reconstruction => Order of spatial convergence = 3.4
2nd degree full base polynomial reconstruction => Order of spatial convergence = 2.9

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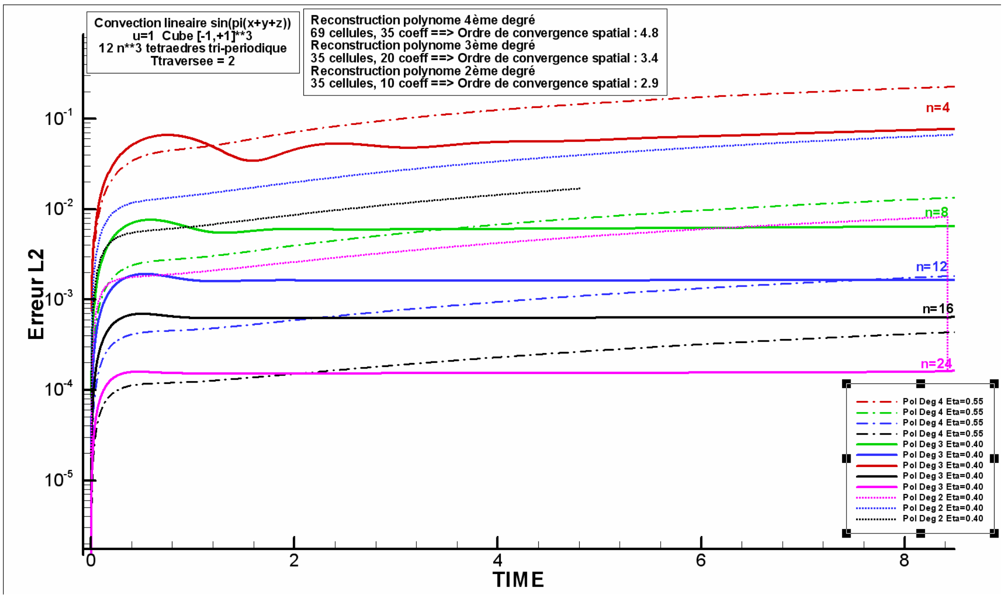



***Figure 1 : Norm of the error in time for the convection of the function sin(x+y+z) in a 3-periodic cube meshed by tetrahedra***

_ Parallel capability

- Loop-based Open-MP programming.

Acceleration of the order of 9 on a dual Westmere board (12 cores, 24 OMP threads) with respect to a single Westmere core.

- No MPI.
- GPU version on NVIDIA TESLA 2070 board (Fortran architecture + Cuda C computational modules): Acceleration of the order 30 with respect to a single Westmere core.

_ Post-processing

- Output in TecPlot ™ Format.
- Internal binary format for visualizations inside the GUI.

2. Case summary

Mach Number 0.1

_ Machines used (number of cores if parallel) : Dual Westmere Board, 12 cores, 24 OMP  threads activated

_ Taubench CPU times on machines used : 7.84s

3. Meshes

Description of meshes used for the case.

Tetrahedral grids obtained by extruding tetrahedra from a regular cartesian grid of cubes.

Each tetrahedron has one edge of the cartesian cube, the opposite edge links the centers of 2 adjacent cubes (see figure 2). This produces 12* n**3 tets. All faces have the same geometry (isocel triangle with sides 1, sqrt(3)/2., sqrt(3)/2.). The cells featuring in the stencils for the LSQ polynomial reconstruction/projection in the convective and diffusive operators are at the same locations in the local reference frame to each interface.

Periodicity assessed at the necessary high space order in all 3 directions.

Case run with 12*48**3, 12*64**3, 12*96**3 , 12*128**3 cells (finest grid, equivalent to 292**3 cells).

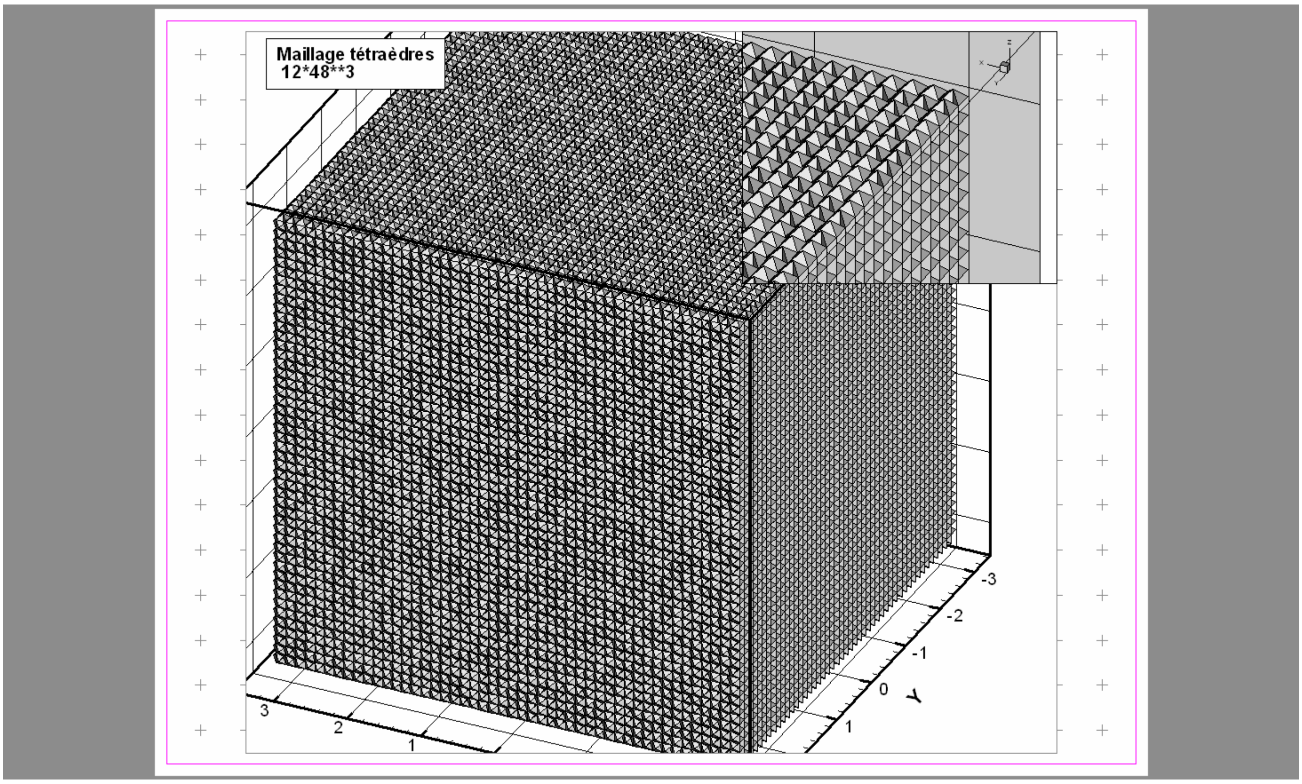



***Figure 2 : Grid of regular tetrahedra***

_ Domain size: the one requested ]-pi,pi[ **3
_Unstructured grid (tetrahedra)_, but with a regular stencil pattern ➔ one only set of interpolation coefficients for the space scheme including the gradient components in the local reference frame to each interface, 18 sets of interpolation coefficients in the global reference frame.

4. Results

| Order of the reconstruction / rows of cells | # of monomials (full 3d base) | # of cells in the biased stencils | # of cells in the union stencil | Weight decay coefficient |
|---|---|---|---|---|
| 2 / 2 | 10 | 20 | 28? | 0.5 |
| 3 / 3 | 20 | 35 | 48 | 0.33 – 0.4 |
| 4 / 4 | 35 | 69 | 90 | 0.25 - 0.33 |

Data for the highest resolution case 12*128**3 (292**3), at maximum reconstruction order (4)

Time step: 1.91 $10^{-3}$ time units (reference time 1. from the convective reference velocity 1. and reference length 1. from the Reynolds number definition )
Total shared memory usage : 11.2 Gbytes
Wall clock time per (explicit) time step (6 RK stages) : 81 s
Total wall clock time (10 000 time steps) : 225 hours

Grid Convergence

Figure 3 presents the results for the enstrophy of runs at a lower resolution : 12*64**3 and 12*96**3 for different orders of polynomial reconstruction. The dotted lines are solutions at higher Reynolds numbers than the one of the test case.
The maxima of enstrophy for the test case (solid lines) are not grid-converged.

Figure 4 presents the time history of the scaled dissipation for the same runs. The curves for the test case data (solid lines) reach the same levels.

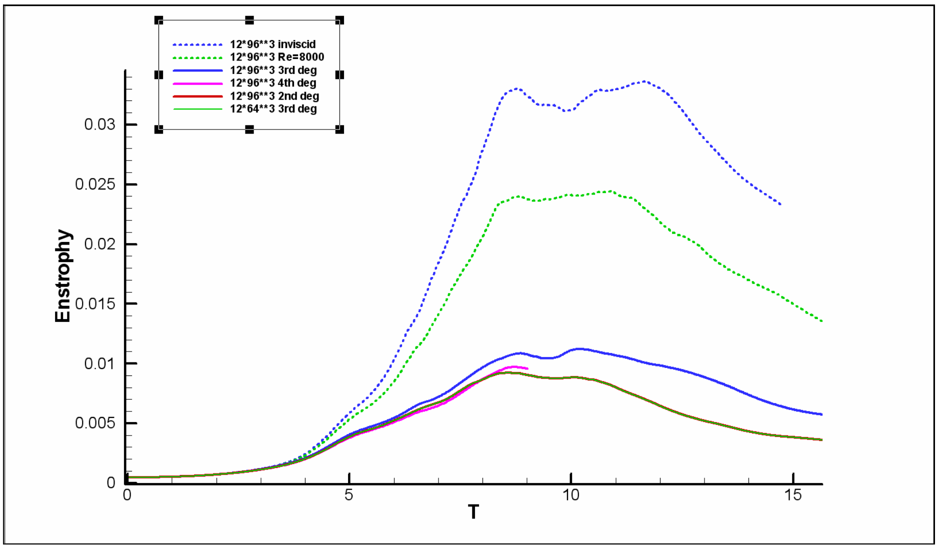



***Figure 4 Enstrophy integral for the C3.5 case and lower viscosity cases***

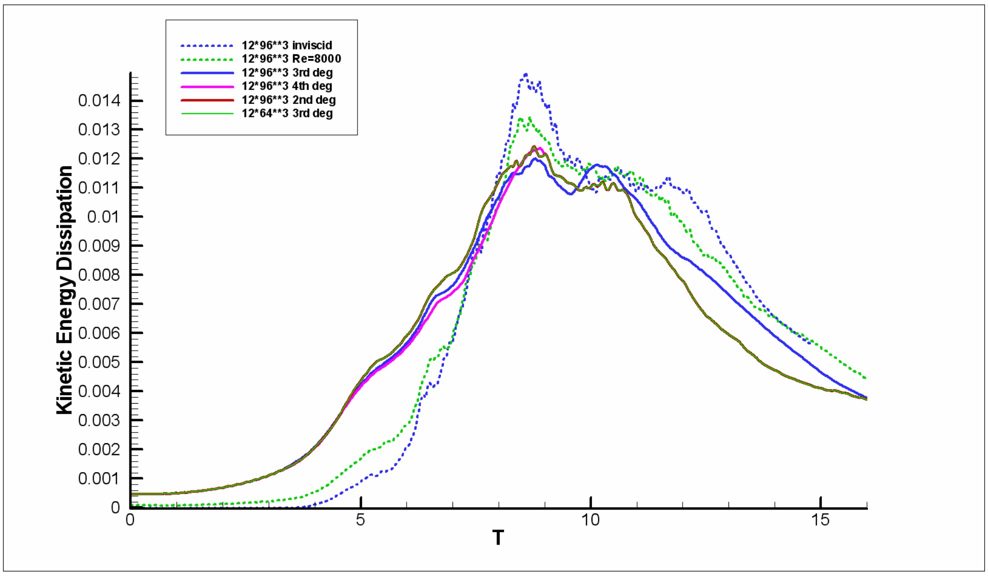



***Figure 4 : Scaled kinetic energy dissipation for the C3.5 case and lower viscosity cases***

On figure 5 are shown the most accurate results : fine grid, high reconstruction order, together with the reference results from a spectral code provided by the coordinator of the workshop test-case. The time history of enstrophy and scaled energy dissipation are on the same plot. The two results from the NXO scheme are almost homothetic.
The variations of time gradients show the same low frequency behavior as in the reference results..

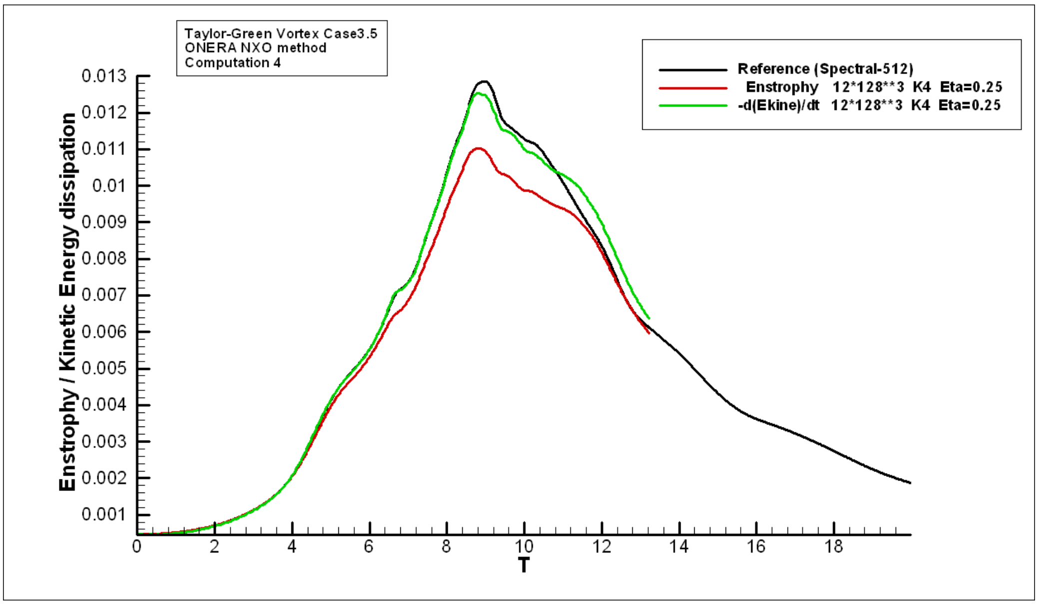



*Figure 5 : Most accurate NXO results shown with the workshop reference results*

On figure 6 are plotted the time derivatives of the preceding curves. The amplitude and phase of the low frequency oscillations are well captured, while a higher frequency is superimposed in the curve of the dissipation derivative. This is representative of an acoustic phenomenon, a complementary case at Mach 1/50 at 12*64**3 resolution shows that it is almost canceled. This phenomenon can be initiated by the fact that the initialisation field prescribed for incompressible flows satifies divU = 0 but not div(rho*U)=0. However this phenomenon is sustained in time, even at later times well after the onset of strong dissipation and not only dependent of the initial state.

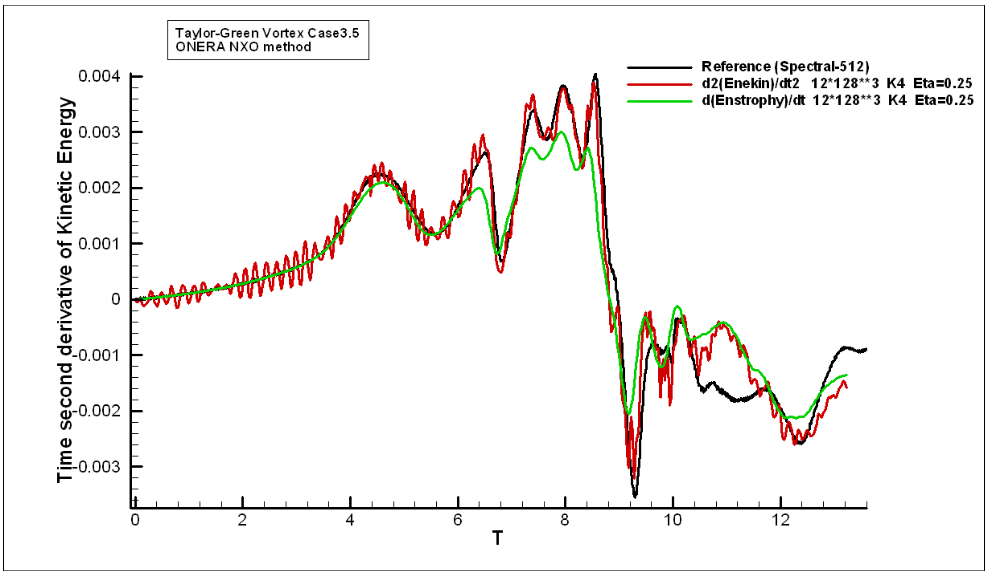
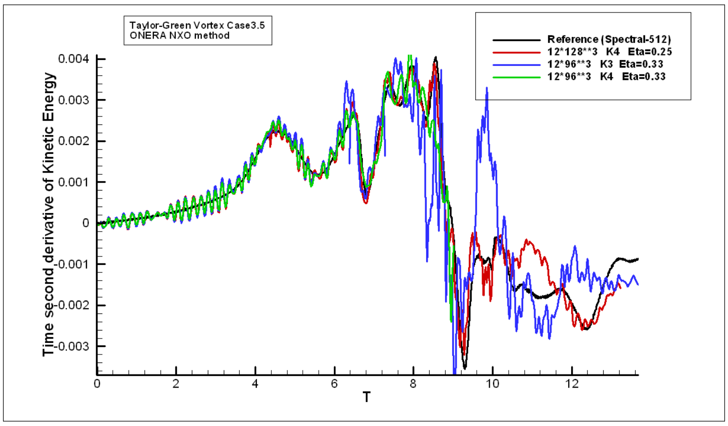
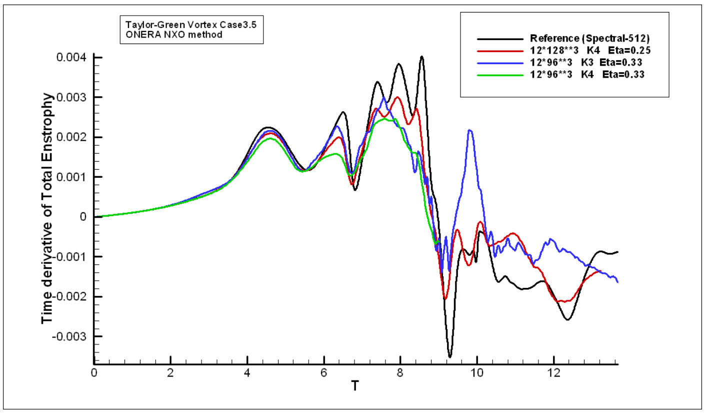



*Figure 6 : Time derivatives of the volume integrals of enstrophy and scaled dissipation*

On figures 7 and 8 are plotted the time derivatives for runs of different grid resolution and scheme accuracy. They show the gain in accuracy necessary to pursue the computation beyond 8s.




***Figures 7 and 8 : Time derivatives of the scaled integral dissipation and of the volume integral of enstrophy***

Figure 9 represents the isolines of vorticity on the symmetry plane at 8s in the most accurate run.

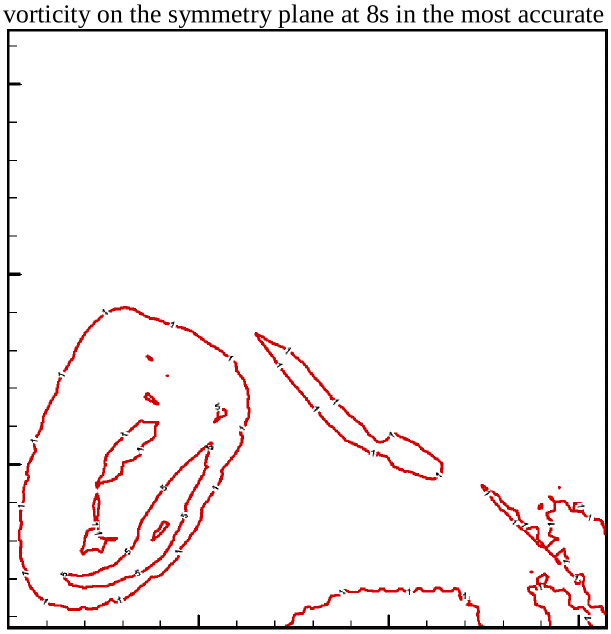


***Figure 9***


Reference 1 : **J.-M. Le Gouez, V. Couaillier, F. Renac**
*High Order Interpolation Methods and Related URANS Schemes on Composite Grids.*
48th AIAA Aerospace Sciences Meeting -Orlando, -USA (04-07 Jan 2010), AIAA-2010-513

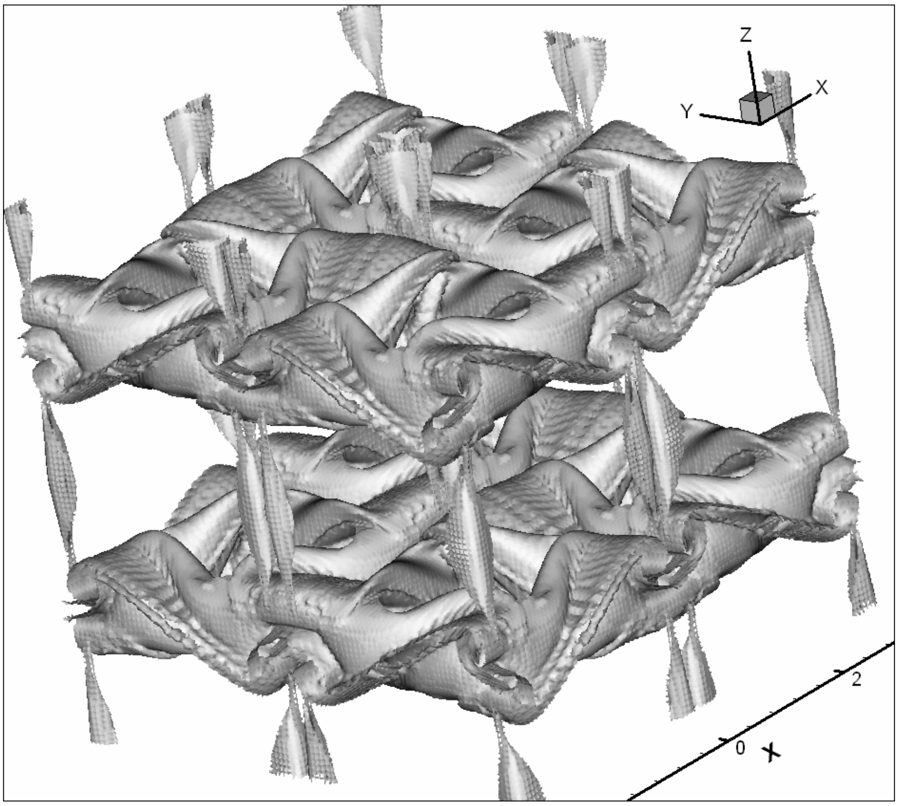



*Figure 10 : Isosurface Enstrophy = 1. before the transition in the run at Reynolds = 8000*

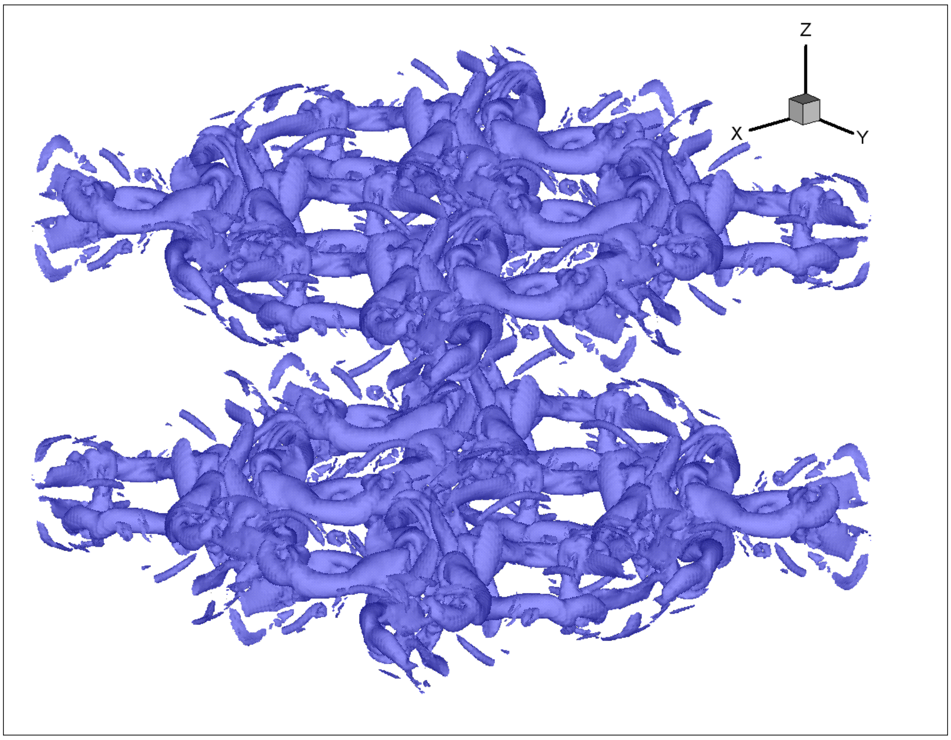



*Figure 11 : Isosurface Enstrophy = 1 at 8s : Case 3.5 run with highest resolution*