C1.2 Ringleb Flow

1. Code description

XFlow is a high-order discontinuous Galerkin (DG) finite element solver written in ANSI C, intended to be run on Linux-type platforms. Relevant supported equation sets include compressible Euler, Navier-Stokes, and RANS with the Spalart-Allmaras model. High-order is achieved compactly within elements using various high-order bases on triangles, tetrahedra, quadrilaterals, and hexahedra. Parallel runs are supported using domain partitioning and MPI communication. Visual post-processing is performed with an in-house plotter. Output-based adaptivity is available using discrete adjoints.

2. Case summary

The default implicit Newton solver was used for all runs in this case. The residual was converged to an absolute L_1 norm below 10^{-10} using a conservative state vector of $\mathcal{O}(1)$ freestream density, velocity, and pressure, and gas constant $R = 1.0$. Runs were performed on the *nyx* supercomputing cluster at the University of Michigan. The number of cores ranged from 1 on the coarsest meshes to 16 on the finest meshes. On one core of the nyx machine, one TauBench unit is equivalent to 16.5 seconds of compute time.

3. Meshes

The meshes were generated using a Matlab script with Q=4 and consist of quadrilateral elements. The boundary conditions are fully described by the given analytical solution that generates isentropic flow properties.

4. Results

The figures and tables below present the results requested for this case with both exact and inviscid-wall boundary conditions along the inner and outer walls. The missing data points correspond to the runs for which the non-linear solver did not converge to tolerance.

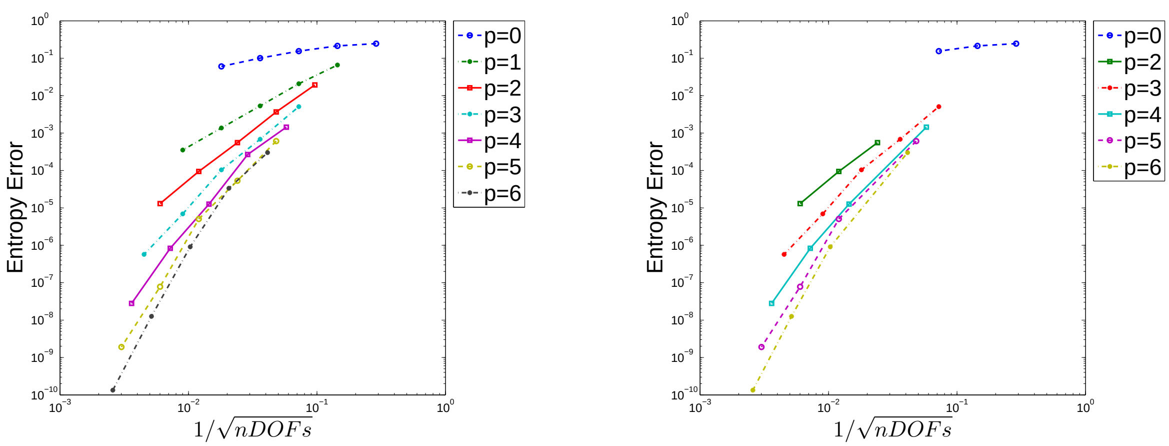

Figure 1: Entropy error convergence with h refinement: exact (left) and inviscid wall (right).

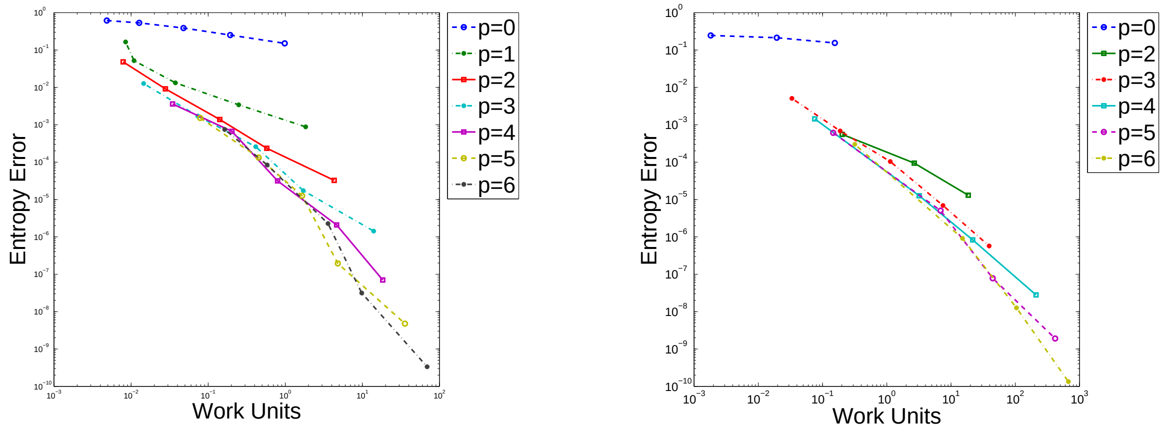

Figure 2: Work units plot: exact (left) and inviscid wall (right).

Table 1: Relevant data for the Ringleb flow case (exact).

Order	Ref=0	Ref=1	Ref=2	Ref=3	Ref=4
$p = 0$	6.1513e-1	5.3353e-1	3.8769e-1	2.5218e-1	1.5109e-1
Rate	-	0.2	0.5	0.6	0.7
Work	4.8485e-3	1.2727e-2	4.7879e-2	1.9273e-1	9.8242e-1
$p = 1$	1.6467e-1	5.2033e-2	1.3284e-2	3.4181e-3	8.7911e-4
Rate	-	1.6	2.0	2.0	2.0
Work	8.4848e-3	1.0909e-2	3.7576e-2	2.4909e-1	1.8376e+0
$p = 2$	4.8328e-2	9.1434e-3	1.3872e-3	2.3496e-4	3.2518e-5
Rate	-	2.4	2.7	2.6	2.9
Work	7.8788e-3	2.7879e-2	1.4121e-1	5.7576e-1	4.3055e+0
$p = 3$	1.2687e-2	1.7059e-3	2.5943e-4	1.7288e-5	1.4254e-6
Rate	-	2.9	2.7	3.9	3.6
Work	1.4545e-2	7.3333e-2	4.1394e-1	1.7139e+0	1.4012e+1
$p = 4$	3.5968e-3	6.6764e-4	3.1477e-5	2.0830e-6	7.0046e-8
Rate	-	2.4	4.4	3.9	4.9
Work	3.4545e-2	2.0364e-1	7.9576e-1	4.6370e+0	1.8368e+1
$p = 5$	1.5213e-3	1.3280e-4	1.2598e-5	1.9528e-7	4.7634e-9
Rate	-	3.5	3.4	6.0	5.4
Work	7.8788e-2	4.5152e-1	1.6552e+0	4.8061e+0	3.5682e+1
$p = 6$	7.5123e-4	8.3767e-5	2.2708e-6	3.1534e-8	3.3490e-10
Rate	-	3.2	5.2	6.2	6.6
Work	1.6364e-1	5.8242e-1	3.5848e+0	9.7933e+0	6.9047e+1

Table 2: Relevant data for the Ringleb flow case (inviscid).

Order	Ref=0	Ref=1	Ref=2	Ref=3	Ref=4
$p = 0$	6.1513e-1	5.3353e-1	3.8769e-1	-	-
Rate	-	0.2	0.5	-	-
Work	1.8182e-3	1.9394e-2	1.5515e-1	-	-
$p = 1$	-	-	-	-	-
Rate	-	-	-	-	-
Work	-	-	-	-	-
$p = 2$	-	-	1.3872e-3	2.3496e-4	3.2518e-5
Rate	-	-	-	2.6	2.9
Work	-	-	2.0242e-1	2.6800e+0	1.8538e+1
$p = 3$	1.2687e-2	1.7059e-3	2.5943e-4	1.7288e-5	1.4254e-6
Rate	-	2.9	2.7	3.9	3.6
Work	3.3333e-2	1.8848e-1	1.1321e+0	7.5042e+0	3.9236e+1
$p = 4$	3.5968e-3	-	3.1477e-5	2.0830e-6	7.0046e-8
Rate	-	-	-	3.9	4.9
Work	7.5758e-2	-	3.2000e+0	2.1773e+1	2.1050e+2
$p = 5$	1.5213e-3	-	1.2598e-5	1.9528e-7	4.7634e-9
Rate	-	-	-	6.0	5.4
Work	1.4667e-1	-	6.8842e+0	4.4596e+1	4.1708e+2
$p = 6$	7.5123e-4	-	2.2708e-6	3.1534e-8	3.3490e-10
Rate	-	-	-	6.2	6.6
Work	3.1758e-1	-	1.5083e+1	1.0445e+2	6.7187e+2

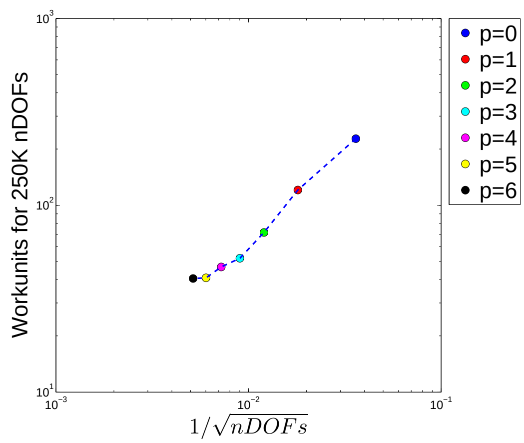

Figure 3: Plot of work units to complete 100 residual evaluations.