# C1.3 Flow over a NACA0012 Airfoil

## 1. Code description

XFlow is a high-order discontinuous Galerkin (DG) finite element solver written in ANSI C, intended to be run on Linux-type platforms. Relevant supported equation sets include compressible Euler, Navier-Stokes, and RANS with the Spalart-Allmaras model. High-order is achieved compactly within elements using various high-order bases on triangles, tetrahedra, quadrilaterals, and hexahedra. Parallel runs are supported using domain partitioning and MPI communication. Visual post-processing is performed with an in-house plotter. Output-based adaptivity is available using discrete adjoints.

## 2. Case summary

The default implicit Newton solver was used for all runs in this case. The residual was converged to an absolute $L_1$ norm below $10^{-7}$ using a conservative state vector of $\mathcal{O}(1)$ freestream density, velocity, and pressure, and gas constant $R = 1.0$. Runs were performed on the *nyx* supercomputing cluster at the University of Michigan. The number of cores ranged from 4 on the coarsest meshes to 64 on the finest meshes. On one core of the nyx machine, one TauBench unit is equivalent to 16.5 seconds of compute time.

## 3. Meshes

The quadrilateral meshes posted on the workshop site, specifically ref0 through ref4, were used for all runs in this case. Prior to the runs, conversion to an equivalent native format was performed without any loss of information.

## 4. Results

The figures and tables below present the results requested for all three conditions in this test case. Output errors were obtained relative to truth solutions computed from adjoint-based $h$-adaptive runs using $p = 2$ on the `ref3` mesh as a starting point.

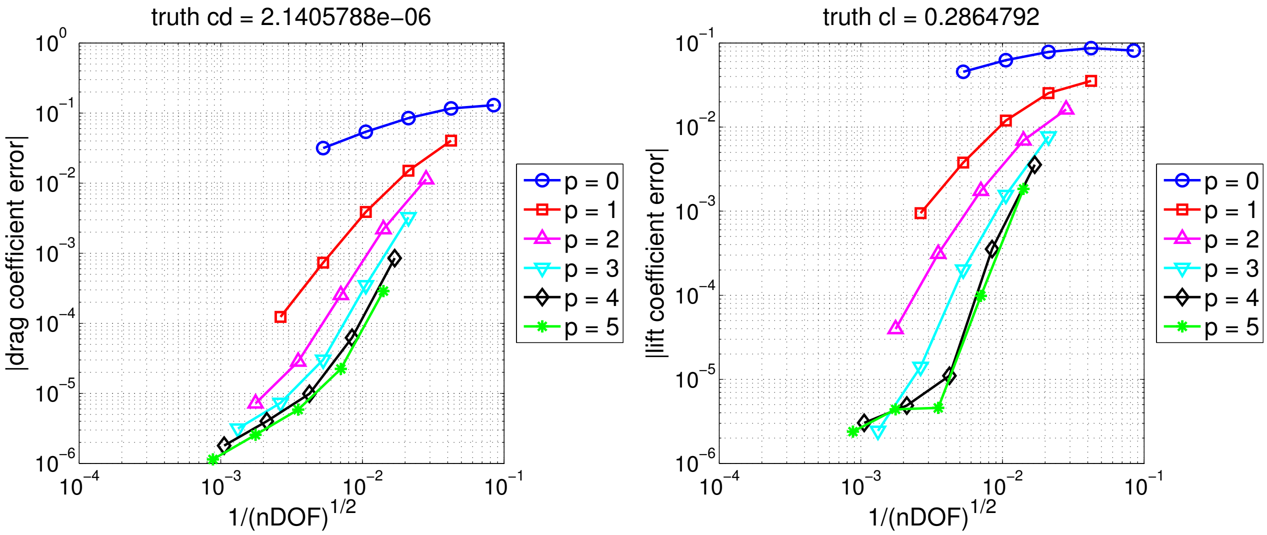

Figure 1: $M = 0.5, \alpha = 2^o$, inviscid: drag and lift error convergence with mesh $h$ refinement.

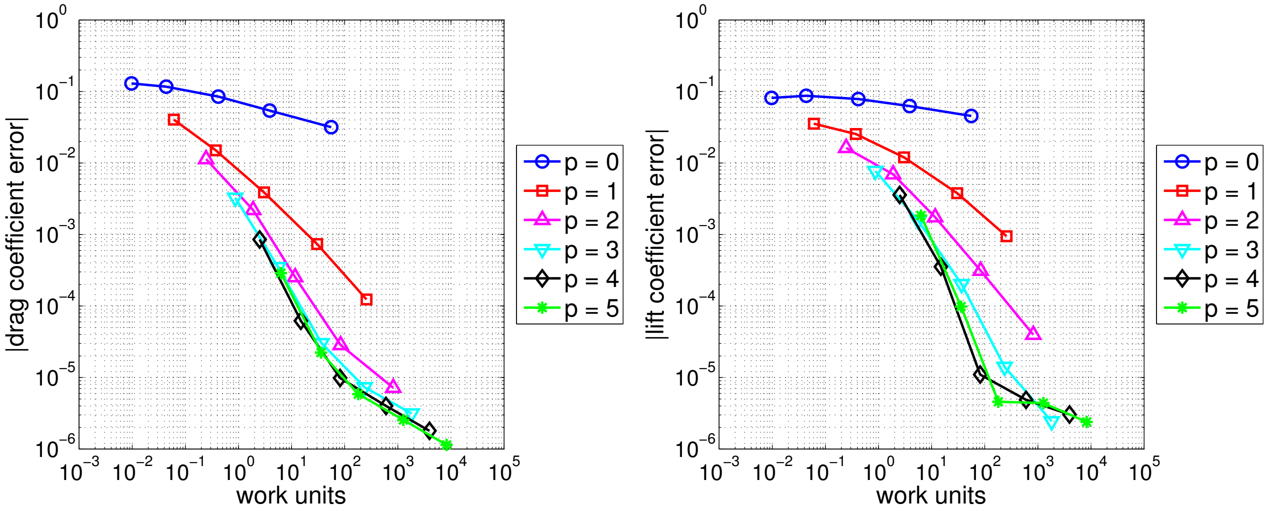

Figure 2: $M = 0.5, \alpha = 2^o$, inviscid: drag and lift error convergence with work units.

Table 1: $M = 0.5, \alpha = 2^o$, inviscid: drag coefficient errors and rates.

| nelem | p = 0 | p = 1 | p = 2 | p = 3 | p = 4 | p = 5 |
|---|---|---|---|---|---|---|
| 140 | 1.2944e-01 | 4.0329e-02 | 1.1330e-02 | 3.2583e-03 | 8.5020e-04 | 2.8801e-04 |
| *rate* | - | - | - | - | - | - |
| 560 | 1.1640e-01 | 1.5031e-02 | 2.1991e-03 | 3.4856e-04 | 6.2168e-05 | 2.2305e-05 |
| *rate* | 0.15 | 1.42 | 2.37 | 3.22 | 3.77 | 3.69 |
| 2240 | 8.4834e-02 | 3.8848e-03 | 2.5312e-04 | 3.0161e-05 | 9.8539e-06 | 5.8536e-06 |
| *rate* | 0.46 | 1.95 | 3.12 | 3.53 | 2.66 | 1.93 |
| 8960 | 5.4132e-02 | 7.3409e-04 | 2.8410e-05 | 7.3092e-06 | 4.0047e-06 | 2.5566e-06 |
| *rate* | 0.65 | 2.40 | 3.16 | 2.04 | 1.30 | 1.20 |
| 35840 | 3.1642e-02 | 1.2344e-04 | 7.1833e-06 | 3.1437e-06 | 1.7959e-06 | 1.1411e-06 |
| *rate* | 0.77 | 2.57 | 1.98 | 1.22 | 1.16 | 1.16 |

Table 2: $M = 0.5, \alpha = 2^o$, inviscid: lift coefficient errors and rates.

| nelem | p = 0 | p = 1 | p = 2 | p = 3 | p = 4 | p = 5 |
|---|---|---|---|---|---|---|
| 140 | 8.1223e-02 | 3.5404e-02 | 1.6228e-02 | 7.7316e-03 | 3.5702e-03 | 1.8399e-03 |
| *rate* | - | - | - | - | - | - |
| 560 | 8.6937e-02 | 2.5367e-02 | 6.9423e-03 | 1.5569e-03 | 3.5403e-04 | 9.8133e-05 |
| *rate* | -0.10 | 0.48 | 1.22 | 2.31 | 3.33 | 4.23 |
| 2240 | 7.8415e-02 | 1.1955e-02 | 1.7437e-03 | 2.0211e-04 | 1.1024e-05 | 4.5766e-06 |
| *rate* | 0.15 | 1.09 | 1.99 | 2.95 | 5.01 | 4.42 |
| 8960 | 6.2518e-02 | 3.7854e-03 | 3.1135e-04 | 1.4070e-05 | 4.8743e-06 | 4.3949e-06 |
| *rate* | 0.33 | 1.66 | 2.49 | 3.84 | 1.18 | 0.06 |
| 35840 | 4.5479e-02 | 9.4633e-04 | 3.9692e-05 | 2.4429e-06 | 3.0366e-06 | 2.3897e-06 |
| *rate* | 0.46 | 2.00 | 2.97 | 2.53 | 0.68 | 0.88 |

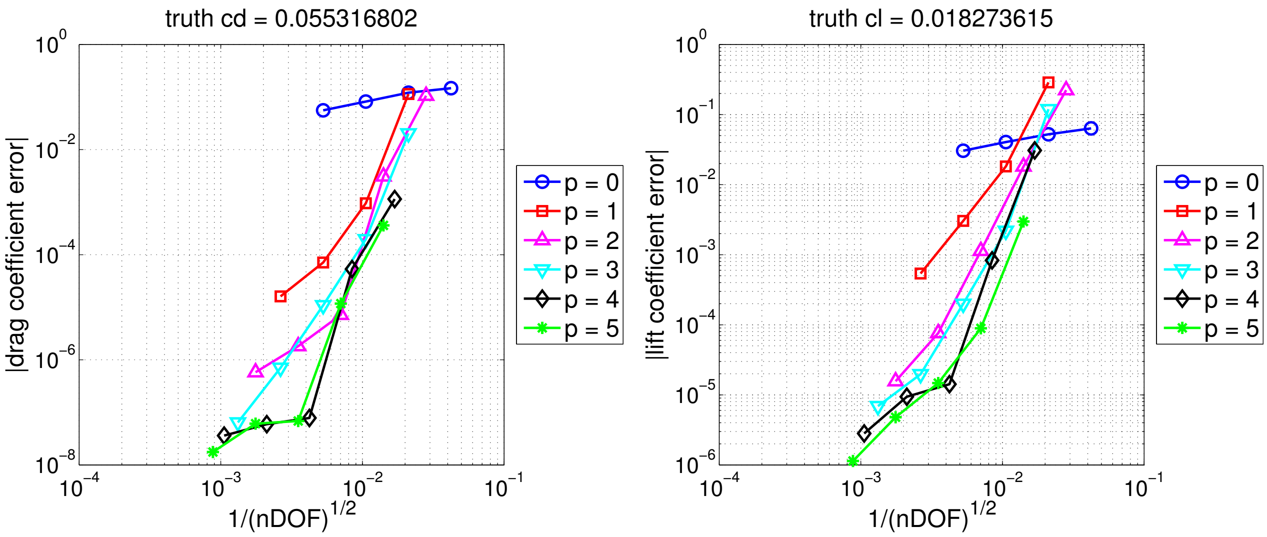

Figure 3: $M = 0.5, \alpha = 1^o, Re = 5000$: drag and lift error convergence with mesh $h$ refinement.

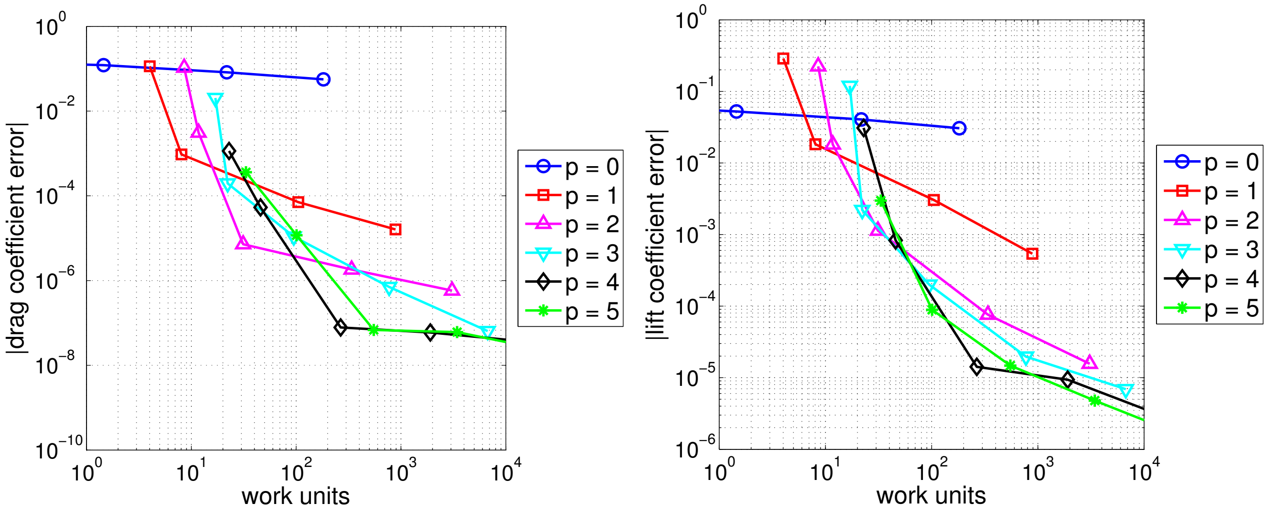

Figure 4: $M = 0.5, \alpha = 1^o, Re = 5000$: drag and lift error convergence with work units.

Table 3: $M = 0.5, \alpha = 1^o, Re = 5000$: drag coefficient errors and rates.

| nelem | p = 0 | p = 1 | p = 2 | p = 3 | p = 4 | p = 5 |
|---:|---:|---:|---:|---:|---:|---:|
| 140 | x | x | 1.0584e-01 | 2.0367e-02 | 1.1423e-03 | 3.6106e-04 |
| *rate* | - | - | - | - | - | - |
| 560 | 1.4717e-01 | 1.1434e-01 | 3.0921e-03 | 1.9646e-04 | 5.3305e-05 | 1.1730e-05 |
| *rate* | - | - | 5.10 | 6.70 | 4.42 | 4.94 |
| 2240 | 1.2103e-01 | 9.5211e-04 | 7.1814e-06 | 1.0918e-05 | 7.8400e-08 | 6.8400e-08 |
| *rate* | 0.28 | 6.91 | 8.75 | 4.17 | 9.41 | 7.42 |
| 8960 | 8.2059e-02 | 7.0890e-05 | 1.8224e-06 | 7.0680e-07 | 5.9600e-08 | 6.1000e-08 |
| *rate* | 0.56 | 3.75 | 1.98 | 3.95 | 0.40 | 0.17 |
| 35840 | 5.5912e-02 | 1.6087e-05 | 5.7940e-07 | 6.4400e-08 | 3.6200e-08 | 1.7600e-08 |
| *rate* | 0.55 | 2.14 | 1.65 | 3.46 | 0.72 | 1.79 |

Table 4: $M = 0.5, \alpha = 1^o, Re = 5000$: lift coefficient errors and rates.

| nelem | p = 0 | p = 1 | p = 2 | p = 3 | p = 4 | p = 5 |
|---:|---:|---:|---:|---:|---:|---:|
| 140 | x | x | 2.2307e-01 | 1.1938e-01 | 3.0813e-02 | 2.9748e-03 |
| *rate* | - | - | - | - | - | - |
| 560 | 6.3572e-02 | 2.8772e-01 | 1.8118e-02 | 2.1970e-03 | 8.2993e-04 | 8.8936e-05 |
| *rate* | - | - | 3.62 | 5.76 | 5.21 | 5.06 |
| 2240 | 5.2277e-02 | 1.8212e-02 | 1.1324e-03 | 1.9937e-04 | 1.4182e-05 | 1.4727e-05 |
| *rate* | 0.28 | 3.98 | 4.00 | 3.46 | 5.87 | 2.59 |
| 8960 | 4.0506e-02 | 3.0369e-03 | 7.5999e-05 | 1.9696e-05 | 9.3876e-06 | 4.7862e-06 |
| *rate* | 0.37 | 2.58 | 3.90 | 3.34 | 0.60 | 1.62 |
| 35840 | 3.0489e-02 | 5.3873e-04 | 1.5648e-05 | 6.9059e-06 | 2.8110e-06 | 1.1370e-06 |
| *rate* | 0.41 | 2.49 | 2.28 | 1.51 | 1.74 | 2.07 |

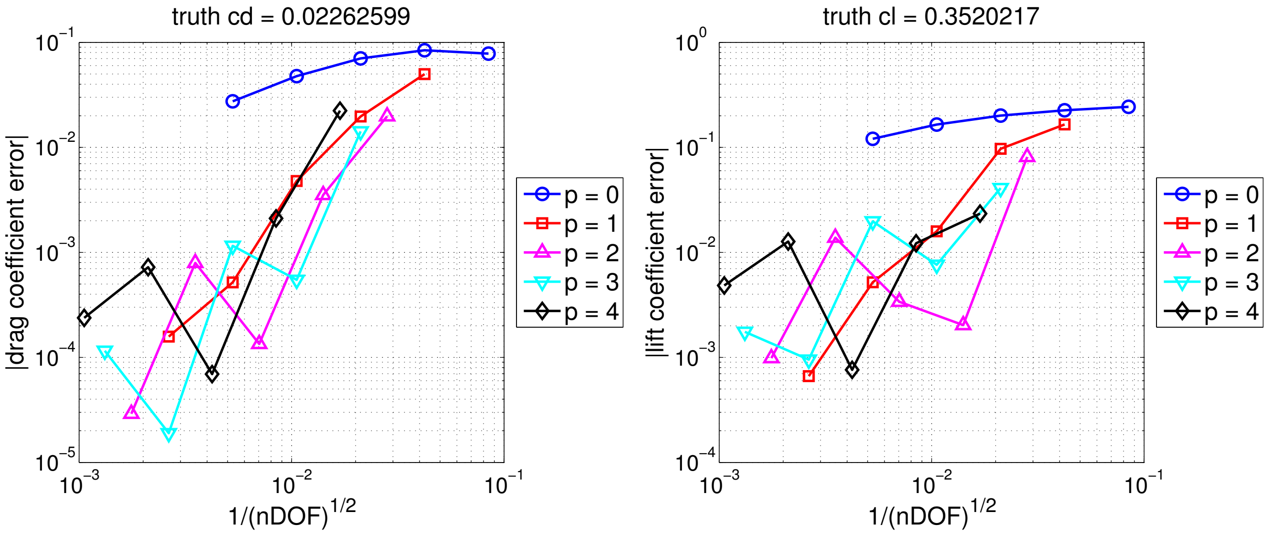

Figure 5: $M = 0.8, \alpha = 1.25^o$, inviscid: drag and lift error convergence with mesh $h$ refinement.

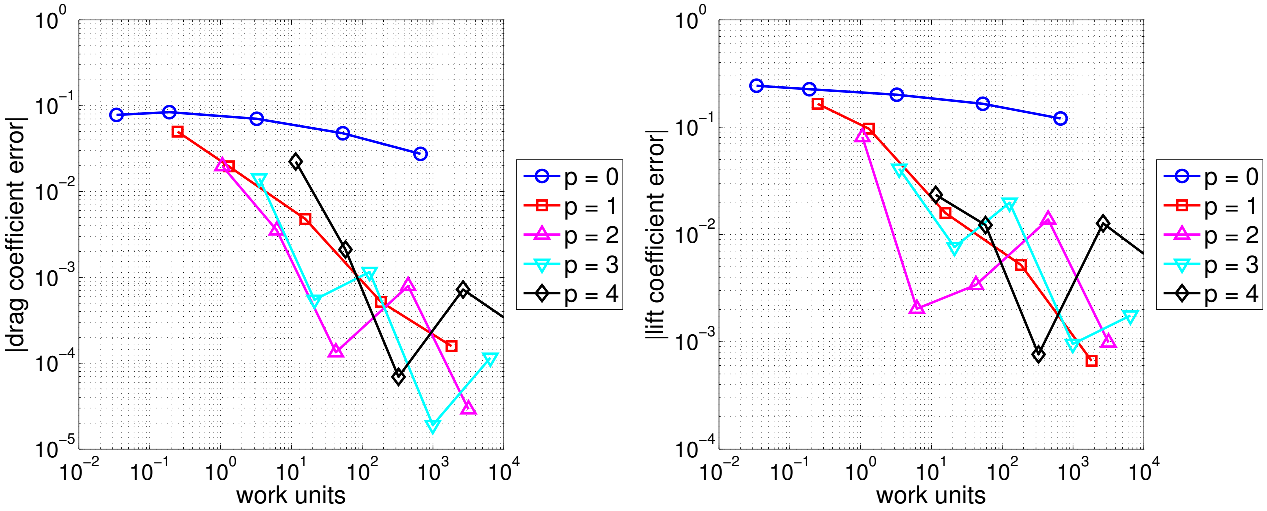

Figure 6: $M = 0.8, \alpha = 1.25^o$, inviscid: drag and lift error convergence with work units.

Table 5: $M = 0.8, \alpha = 1.25^o$, inviscid: drag coefficient errors and rates.

| nelem | p = 0 | p = 1 | p = 2 | p = 3 | p = 4 |
|---:|---:|---:|---:|---:|---:|
| 140 | 7.8128e-02 | 4.9882e-02 | 1.9769e-02 | 1.4211e-02 | 2.2356e-02 |
| *rate* | - | - | - | - | - |
| 560 | 8.4119e-02 | 1.9667e-02 | 3.5247e-03 | 5.4782e-04 | 2.1089e-03 |
| *rate* | -0.11 | 1.34 | 2.49 | 4.70 | 3.41 |
| 2240 | 7.0357e-02 | 4.7830e-03 | 1.3418e-04 | 1.1538e-03 | 6.9367e-05 |
| *rate* | 0.26 | 2.04 | 4.72 | -1.07 | 4.93 |
| 8960 | 4.7728e-02 | 5.1836e-04 | 7.9317e-04 | 1.8985e-05 | 7.2221e-04 |
| *rate* | 0.56 | 3.21 | -2.56 | 5.93 | -3.38 |
| 35840 | 2.7483e-02 | 1.5776e-04 | 2.9041e-05 | 1.1569e-04 | 2.3818e-04 |
| *rate* | 0.80 | 1.72 | 4.77 | -2.61 | 1.60 |

Table 6: $M = 0.8, \alpha = 1.25^o$, inviscid: lift coefficient errors and rates.

| nelem | p = 0 | p = 1 | p = 2 | p = 3 | p = 4 |
|---:|---:|---:|---:|---:|---:|
| 140 | 2.4339e-01 | 1.6528e-01 | 8.0970e-02 | 4.0942e-02 | 2.3411e-02 |
| *rate* | - | - | - | - | - |
| 560 | 2.2569e-01 | 9.6853e-02 | 2.0307e-03 | 7.6237e-03 | 1.2223e-02 |
| *rate* | 0.11 | 0.77 | 5.32 | 2.43 | 0.94 |
| 2240 | 2.0075e-01 | 1.5850e-02 | 3.3836e-03 | 1.9762e-02 | 7.6152e-04 |
| *rate* | 0.17 | 2.61 | -0.74 | -1.37 | 4.00 |
| 8960 | 1.6507e-01 | 5.1934e-03 | 1.3806e-02 | 9.5308e-04 | 1.2683e-02 |
| *rate* | 0.28 | 1.61 | -2.03 | 4.37 | -4.06 |
| 35840 | 1.2060e-01 | 6.6332e-04 | 9.8357e-04 | 1.7535e-03 | 4.8419e-03 |
| *rate* | 0.45 | 2.97 | 3.81 | -0.88 | 1.39 |