Summary of the C2.4 test case results

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Test case C2.4

Laminar flow at $M = 0.3$, $Re = 4000$ and $\alpha = 12.5^\circ$ around the delta wing

Reference values (taken from [LH10]):
$C_d^{\text{ref}} = 0.1658$, $C_l^{\text{ref}} = 0.347$.

[Mach number isosurfaces (left) and slices (right) [LH10]]

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Following nested hexahedral meshes have been provided on the workshop (hiocfd) homepage

- delta.1.msh with 408 cells
- delta.2.msh with 3,264 cells
- delta.3.msh with 26,112 cells
- delta.4.msh with 208,896 cells

“Official” convergence criterion (taken from “Notes for all participants”)
Reduction of the density residual to $10^{-10}$ relative to freestream conditions measured in a normalized $L^2$-norm, i.e.

$$\frac{R}{R_\infty} < 10^{-10} \quad \text{for} \quad R = L^2(\text{Res}_\varrho) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \text{Res}_{\varrho,i}^2}$$
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Data available from
- Alessandro Colombo, Francesco Bassi, University of Bergamo, UBerg
- Krzysztof Fidkowski, University of Michigan, UMich
- Ralf Hartmann, DLR, Braunschweig, DLR
- Li Wang, J. Taylor Erwin, W. Kyle Anderson, University of Tennessee, UTenn
# Summary of test case C2.4

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## Case details

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<th>U Berg</th>
<th>U Mich</th>
<th>DLR</th>
<th>U Tenn</th>
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<td>discretization</td>
<td>DG, BR2</td>
<td>DG, BR2</td>
<td>DG, BR2</td>
<td>DG, SIPG</td>
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<tr>
<td>basis functions</td>
<td>$P_p(\kappa)$, orth.norm.</td>
<td>$\sigma(Q_p(\hat{\kappa}))$</td>
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<td>$\sigma(P_p(\hat{\kappa}))$</td>
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<tr>
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<td>Roe</td>
<td>Roe+entropy fix</td>
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<tr>
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<td>Roe + $u_\infty$?</td>
<td>characteristic</td>
<td>subsonic in/outflow</td>
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<tr>
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<td>bw Euler</td>
<td>Newton</td>
<td>Newton</td>
<td>p-MG+el.GS/GMRES</td>
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<tr>
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<td>grids (hiocfd)</td>
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<td>grids (hiocfd)</td>
<td>unstruct./tretrah.</td>
</tr>
<tr>
<td>convergence criterion</td>
<td>official $R/R_\infty &lt; 10^{-10}$</td>
<td>other $R &lt; 10^{-7}$</td>
<td>official $R/R_\infty &lt; 10^{-10}$</td>
<td>other $R &lt; 10^{-11}$</td>
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<tr>
<td>wall</td>
<td>adiabatic (instead of isothermal)</td>
<td></td>
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</table>

**Note**, that work units of UMich & UTenn are not comparable to others.
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<td>$\sqrt{\frac{1}{N} \sum_i^N Res^2_{\rho,i}}$?</td>
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Reference values

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  grids (hiocfd), isothermal wall boundary condition
- obtained by UTenn(DG): $C_d^{\text{ref}} = 0.1605$, $C_l^{\text{ref}} = 0.3455$
  unstructured grids, adiabatic wall boundary condition
Test case C2.4: $C_d$ vs. $h$

Summary of the C2.4 test case results

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Test case C2.4: $C_l$ vs. $h$

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Test case C2.4: Error in $C_d$ vs. $h$

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Test case C2.4: Error in $C_d$ vs. $h$ for $p = 2$
Test case C2.4: Error in $C_l$ vs. $h$

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Test case C2.4: Error in $C_l$ vs. $h$ for $p = 2$

Summary of the C2.4 test case results  
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Test case C2.4: Error in $C_d$ vs. workunits

Summary of the C2.4 test case results

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Test case C2.4: Error in $C_l$ vs. workunits
Test case C2.4: Summary

- Comparison of results of **UTenn** with the results of the other partners is difficult as **UTenn** computed on different grid sequence with different boundary conditions and different convergence criterion.

- Very good agreement of \( C_d \) and \( C_l \) values between **UMich** and **DLR** due to the use of very similar numerical schemes.

- However, **UMich** required a factor of 7–70 more work units than **DLR**.

- Difference due to parallelization issues and/or different convergence criterion?

- Difference of results between **UBerg** and **UMich**/**DLR** due to different basis functions/discrete function spaces.

- There is some agreement when comparing results for similar dimension of discrete function spaces instead of for the same polynomial degree.

- Due to the non-smoothness of the flow solution there is not much gain (if at all) in computing time using a global \( p \) larger than 2.
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